Deception and Competition in Search Markets*

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Abstract

We study the interplay between deception and consumer search in a search market where firms may deceive some naive consumers with inferior products that display hidden (bad) attributes. We derive an equilibrium in which both superior and inferior quality is offered and show that as search frictions vanish, superior goods are entirely driven out of the market. Deception harms sophisticated consumers, as it forces them to search longer to find a superior product. We argue that policy interventions that reduce search frictions such as the standardization of price and package formats may harm welfare. In contrast, reducing the number of naive consumers through transparency policies and education campaigns as well as a minimum quality standard and a price floor regulation can improve welfare.

Keywords: Deceptive product, Inferior product, Naivete, Consumer Search

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1 Introduction

In many markets, products are complex, and consumers need a basic level of (financial, legal, technical, etc.) expertise to evaluate and compare products. This implies that sellers can deceive and exploit "naive" consumers who lack the necessary expertise, for example, by hiding (bad) attributes or the true future usage costs of their products. Starting with Gabaix and Laibson (2006) there is by now a large literature that studies firms' incentives to engage in deception (see the literature review below). In this article, we address a novel issue: the interplay between deception and consumer search. This is motivated by the observation that even for "sophisticated" consumers, who possess the relevant expertise, evaluating and comparing complex products is time-consuming and involves costly search.

Explicitly accounting for consumer search delivers new and economically relevant insights. Most strikingly perhaps, we show that stronger competition in the form of smaller search frictions may harm overall welfare, as it increases firms' incentives to engage in inefficient deception. We also identify a novel channel through which deception harms sophisticated consumers: they waste more time examining irrelevant inferior products. These findings have implications for the evaluation of policy measures that aim at lowering search frictions such as the harmonization of price or product formats, or transparency policies that seek to educate naive consumers.[1] We also discuss other policy measures and show, for example, that imposing price floors may enhance welfare.

An important application are markets for financial services. For instance, credit cards often display steep interest rate changes, current accounts feature high overdraft charges, often hidden in fine print, and mortgage loans are advertised with low introductory teaser rates, while hiding high reset rates.[2] These products tend to generate enormous profits at the expense of few vulnerable consumers which is difficult to reconcile with conventional models of consumer behavior.[3] At the same time, evidence suggests that search frictions are important in markets for financial services, as most consumers do not engage in substantial search, but visit only very few competitors. For example, Lee and Hogarth (2000) reports that about one-fourth (23%) of US mortgage borrowers contact only one lender, and the median number of lenders contacted is

[1] For example, the Office of Fair Trading discusses in the context of the UK retail banking market the harmonization of price and product formats in order to address "the difficulties caused by lack of information (which) are exacerbated by the fact that different banks use different terminology and present interest rates and charges in different ways".

[2] See, for example, Ausubel (1991), Stango and Zinman (2009) and Stango and Zinman (2014) on credit cards, OFT (2008) on current accounts, and Woodward and Hall (2010) and Gurun et al. (2016) on mortgages.

[3] See Armstrong and Vickers (2012) and OFT (2008) on the UK retail banking market where insufficient fund charges made up 30% of the total revenue generated by current accounts and were incurred by only 23% of all customers.

three. Another example is the UK retail banking market where, according to OFT (2008), only $15-20\,\%$ of all consumers make explicit comparisons across products in terms of interest and rates, although $69\,\%$ agree that there may be better alternatives available in theory.[4] This reflects that studying financial products and examining how they fit one's needs requires time and effort, and is perceived as tedious and annoying.

To capture the interplay between deception and search, we extend a standard model of consumer search and allow for the presence of naive consumers who are susceptible to deception.[5] A firm engages in deception when it offers a product of inferior quality with hidden (bad) attributes, opaque usage costs, or surcharge fees concealed in the fine print. Alternatively, a firm can choose to be "candid" by offering a product of superior quality. Sophisticated consumers engage in costly search to detect the price, the quality, and the consumer-specific fit of any product. Naive consumers, in contrast, lack the skills to evaluate product quality and fit, and thus fail to distinguish inferior from superior products. In effect, they search for attractive prices only.[6,7] Since inferior products are assumed to be socially inefficient, in the benchmark without any naive consumers, competition ensures that all firms are candid and only offer superior products.

In contrast, when some consumers are naive, deceptive and candid firms co–exist in equilibrium, entailing market segmentation. Sophisticated consumers never buy inferior products but search until they find a suitable superior product. Naive consumers purchase from the first firm they visit provided its price is in an acceptable range. We show that, in equilibrium, deceptive and candid firms charge similar prices. This entails that a firm's product choice is driven by the trade-off between selling the (high cost) superior product at a low mark–up to all consumers and selling the (low cost) inferior product at a high mark–up to only naive consumers.

Our insight that lower search frictions may have a detrimental effect on the provision of quality in the market is driven by the fact that lower search frictions affect a firm's trade-off in favor of becoming deceptive. Intuitively, as search costs get small, sophisticated consumers can compare

[4]For further evidence on the importance of search in financial service markets, see also Mayer and Pence (2008) and Scharfstein and Sunderam (2013) on mortgages, or Sirri and Tufano (1998) and Hortacsu and Syverson (2004) on mutual funds.

[5]In the absence of naive consumers, our model essentially corresponds to Wolinsky (1986) or Anderson and Renault (1999).

[6]For instance, in the context of a mortgage market, this would mean that a naive consumer searches and buys on the basis of teaser rates alone.

[7]More precisely, we model naive consumers as analogy-based reasoners in the sense of Jehiel (2005). This means that a naive consumers, while having correct beliefs about the marginal distributions of prices and qualities in the market, fail to understand the correlation between price and quality. In particular, they fail to correctly infer on quality from observed prices. Another interpretation which is consistent with our modeling choice is that naive consumers are unaware of hidden product features and consequently make no (correct) inference by assumption.

firms essentially for free. This eliminates a candid firm's mark–up and profit ultimately leading all firms to adopt the deceptive business model which guarantees positive profits due to high mark-ups.[8] Thus, if only a small fraction of consumers is vulnerable to deception, intense competition in the form of small search costs has the striking consequence that the market will supply almost only inferior quality.

The effect of smaller search costs on the share of candid firms in the market is clear-cut as search costs vanish, because candid mark–ups entirely erode in this case. For non-negligible search costs, there are counter-veiling effects, because what matters for the comparative statics is not only the change of mark–ups but also the absolute size of demand across the two segments: As search costs fall, the difference in mark–ups between candid and deceptive firms shrinks which makes the candid business model with the lower mark–up more attractive. On the other hand, all else equal, mark–ups in both segments fall which harms candid firms more due to their larger demand.[9] We identify a necessary and sufficient condition for profits in the candid segment to decrease more than in the deceptive segment in response to a marginal decrease in search costs. In this case, lower search frictions reduce the fraction of candid firms in equilibrium.[10]

This finding implies that stiffer competition may be undesirable from a welfare point of view, because the efficiency losses from the increased consumption of inefficient inferior products by naive consumers may outweigh the gains due to savings on search costs. If this is the case, then the ensuing welfare gains are, however, unequally distributed among market participants: Sophisticated consumers lose out, because they are harmed by higher prices and search costs, whereas firms gain, and naive consumer welfare goes up if the gains due to the higher average quality consumed outweigh the losses due to higher prices and search costs.

[8] As we explain in the main text, this intuition is somewhat incomplete because what matters for sophisticated consumers is not nominal, but effective search costs, that is, the costs it takes to find not any, but a candid firm. If the number of candid firms shrinks, then, all else equal, sophisticated consumer waste time evaluating deceptive products which they will not end up buying. As we show, effective search costs are monotone in nominal search costs and, in fact, go to zero if nominal search costs do, although the share of candid firms vanishes.

[9] The equilibrium effect of an increase in search costs on the candid mark-up is, however, not clear cut. On the one hand, as in standard search models, as search costs fall, sophisticated consumers search more, so that, all else equal, candid firms reduce their prices in response. On the other hand, in our framework, the number of candid firms changes endogenously with search costs which affects the composition of naive and sophisticated customers that make up a firm's demand, possibly resulting in higher or lower prices. As we show, whenever the share of candid firms drops as search costs fall, then equilibrium prices go down. (Prices in our setting may however also increase as search costs fall.)

[10] This result is qualitatively consistent with Ellison and Ellison (2009) who documents that in response to the better search technology (in particular for prices), firms in the online market for memory modules began to "bundle low-quality goods with unattractive contractual terms, like providing no warranty and charging a 20 % restocking fee on all return."

Our main results thus suggest that policy interventions that aim at reducing search frictions may be socially undesirable. This may include policies that facilitate the comparability of products, such as requiring firms to use standardized price, package, and product information formats, or the promotion of online marketplaces that facilitate the exchange of price information. In contrast, we argue that minimum quality standards, or, more unconventionally, imposing a price floor may improve welfare in our setting.[11] Intuitively, a price floor may impede the downward spiral in prices which makes the candid business model unviable. Despite the price increase, a price floor regulation may even increase consumer welfare if the difference in quality between superior and inferior products is sufficiently large.

Our analysis also sheds new light on the effects of information and education campaigns that create consumer awareness and reduce consumer naivete (such as promoting financial literacy in the context of financial service markets). As we show, this increases welfare (under a plausible sufficient condition), because as a result of a shrinking naive customer base, fewer firms adopt the deceptive business model. In our search framework, this benefits also sophisticated consumers, as they waste less time examining inferior products. In this sense, naive consumers exert a negative externality on sophisticated consumers in our framework.

On a related note, we contribute to the debate whether firms themselves have incentives to turn naive into sophisticated consumers ("unshrouding"). We argue that the answer depends on whether firms can educate only consumers who visit their own store or reach all consumers in the market and educate them through advertising. In the latter case, unshrouding is profitable for each candid firm, because candid firms steal the entire business from deceptive firms and earn positive profits with any consumer. On the other hand, neither a candid nor a deceptive firm has an incentive to educate naive consumers who visit its store, because this could only trigger them to search for a better firm instead. Moreover, the more consumers are naive, the higher are industry profits so that reducing the number of naive consumers reduces profits in the "long-run".

Related Literature

Our paper is related to a literature in behavioral industrial organization which studies how firms can exploit boundedly rational consumers. Most closely related are papers which study markets where firms offer goods with (unavoidable) future add-on services which naive consumers fail to anticipate, and show that deception can occur in equilibrium even under Bertrand competition (see, e.g., Gabaix and Laibson (2006), Spiegler (2006), Armstrong and Chen (2009), Armstrong and Vickers (2012), Heidhues et al. (2017)). We contribute to this literature by adding consumer search to the picture, allowing novel dimensions of comparative statics, in particular with respect

[11] More precisely, we consider the effect of an exogenously imposed (marginal) price increase relative to the equilibrium price.

to search frictions, which gives rise to new policy implications with respect to the harmonization of price formats and price floors. A prominent theme in the literature is how the number of naive consumers affects (the division) of welfare. Our paper identifies a novel adverse spillover effect from naive to sophisticated consumer welfare: Because the presence of naive consumers reduces the number of candid firms, sophisticated consumers have to search longer to find them. Finally, our analysis of firms' shrouding incentives differs from the literature because in our framework, sophisticated consumers are not subsidized by naive ones and it allows to distinguish different unshrouding strategies.

A closely related paper is also Piccione and Spiegler (2012) whose analysis, like ours, implies that policy interventions that aim at facilitating the comparability of products can turn out to harm consumer welfare. The underlying reason is, however, rather different. In Piccione and Spiegler (2012), a consumer' ability to make a price comparison depends on the price formats chosen by firms. Firms face a trade-off between maximizing sales volume by combining low prices with easily comparable price formats and maximizing mark–ups by combining high prices with onerously comparable formats. Piccione and Spiegler (2012) shows that ("local") interventions that make relatively easily comparable formats even more easily comparable may backfire because firms ar more likely to adopt onerously comparable formats as an equilibrium response. As a result, average prices increase. If, on the other hand, the intervention is "global" and makes all formats more easily comparable, then Piccione and Spiegler (2012) shows that the intervention is desirable. In contrast, in our model, prices may drop if they become more easily comparable, and what makes the intervention undesirable is that the quality in the market deteriorates, suggesting competing testable predictions. Also, in our framework, interventions that ease comparability (i.e., reduction in search costs) are "global" by definition. Finally, the underlying consumer bias is different. Whereas consumers in Piccione and Spiegler (2012) have difficulties in comparing prices, they have difficulties to assess a product's quality in our setting, suggesting different domains of applicability.[12]

Our paper also contributes to the literature on consumer search by integrating naive consumers and seller deception into the seminal papers by Wolinsky (1986) and Anderson and Renault (1999). While in this literature, lowering search frictions is typically considered socially

[12]To the extent that seemingly pro–competitive interventions may backfire, our paper is also related to Varian (1980)'s model of sales where firms set prices so as to trade off attracting consumers who compare prices across firms and reaping loyal customers who do not compare prices. An intervention which leads to a higher number of competitors may backfire because it makes reaping loyal consumer relatively more profitable, entailing an average price increase. Also in Vickers et al. (2009), the seemingly consumer-friendly introduction of a price cap may actually be harmful such that consumers pay higher prices on average, as it reduces their incentives to search.

desirable,[13] our key point is that lower search costs may be detrimental because it enhances a firm's incentives to become deceptive. A related point is made in Kuksov (2004) who argues that lower search costs may give rise to socially excessive product differentiation incentives.[14]

Our finding that prices may decrease in search costs echoes similar findings in the literature.[15] In our model, the price non-monotonicity originates in the fact that the share of candid firms is endogenous which affects both effective search costs and the composition of demand for an individual firm. Somewhat similarly, in Moraga-González et al. (2017) lower search costs also alter the composition of demand, but in their case, this is due to the market entry of less versed consumers whose demand is less elastic.

Our paper is finally related to work which considers consumers who rationally decide to remain "naive" about the quality of the products they purchase. In Gamp (2016b), consumers trade off the risks of a bad buy with the savings on search costs, but in contrast to the current article, always detect larger scams so that, ultimately, equilibria without deception exist. Similarly, in Heidhues et al. (2017), consumer rationally divide their attention between comparing prices across firms ("browsing") and learning the details of few products in depth ("studying"). As our paper, they argue that a minimum quality standard (such as regulating additional prices and secondary product features) improves consumer welfare, among other things, because it leads to more browsing which enhances competition and benefits consumers.

This paper is organized as follows. The next section introduces the model. Section 3 derives equilibrium conditions. Section 4 and 5 perform comparative statics. Section 6 discusses firms' incentives to unshroud, and section 7 discusses policy implications. Section 8 discusses extensions, and section 9 concludes. All proofs are in the appendix.

2 Model

We consider a search market with a unit mass of firms indexed by $k \in [0,1]$ and a unit mass of consumers indexed by $i \in [0,1]$. Each consumer seeks to purchase at most one good. Goods are differentiated, and consumer i's utility from purchasing the good offered by firm k is equal to

$$u_{ik} = q_k + \theta_{ik} - p_k, \tag{1}$$

[13]See, e.g., Bakos (1997), Anderson and Renault (1999), Cachon et al. (2008), Larson (2013), Bar-Isaac et al. (2012), or Fishman and Levy (2015).

[14]Related, it has also been observed that it can be welfare improving if firms may raise search costs through obfuscation (see Gamp (2016a) and Taylor (2017)).

[15]See, e.g., Janssen and Moraga-González (2004), Armstrong and Zhou (2011), Janssen and Shelgia (2015), Moraga-González et al. (2017), or Choi et al. (2017).

where q_k is the "quality" offered, and p_k is the price chosen by firm k. The term θ_{ik} is a consumer-firm specific match-value which represents idiosyncratic product fit. It is common knowledge that for all i, k, the associated random variable θ_{ik} is distributed on the support $[\underline{\theta}, \overline{\theta}]$ with (sufficiently smooth) cumulative density function F and mean zero (hence $\underline{\theta} < 0$), independent and identical across consumers and firms. We assume that the corresponding probability density function f is log-concave with $f(\overline{\theta}) > 0$ which implies that F has an increasing and unbounded hazard rate

$$h(\theta) \quad = \quad \frac{f(\theta)}{1-F(\theta)}. \tag{2}$$

The objective of our analysis is to study market outcomes when some consumers lack the ability to assess products and can be deceived about the true product characteristics. To capture this, we assume that firms can offer either a good of superior quality $\overline{q}$ or of inferior quality $\underline{q}$. As explained in more detail below, the inferior product is "deceptive" in the sense that some consumers cannot distinguish it from a superior product (for example, due to hidden (bad) attributes or opaque contract terms). The quality of a product determines its production costs with $c(q)$ being the marginal costs for producing a good with quality q. Let $\underline{c} = c(\underline{q})$ and $\overline{c} = c(\overline{q})$, with $\underline{c} < \overline{c}$. Moreover, let $\Delta c = \overline{c} - \underline{c}$ and $\Delta q = \overline{q} - \underline{q}$. To make the problem interesting, we assume that the superior good is socially efficient,

$$\Delta q > \Delta c, \tag{3}$$

and that consumers, net of prices, prefer a superior over an inferior product even if the former displays the worst and the latter the best product fit:

$$\Delta q > \overline{\theta} - \underline{\theta}. \tag{4}$$

To purchase a product from a firm, a consumer has to visit it and examine its product which entails a (search) cost. A consumer may examine several products, one at a time and in a random order so as to extend his choice set of products. Prior to examining a product, a consumer is uninformed about its price p_k and its characteristics (q_k, θ_{ik}). To capture that some consumers lack the ability to fully understand all product characteristics and are susceptible to deception, we assume that there is a fraction ν^N of "naive" consumers. Upon visiting a firm k, a naive consumer only learns the price p_k but neither observes the quality q_k nor the product fit θ_{ik}.[16] We assume that a naive consumer fails to draw the correct inference from observed prices. More precisely, a naive consumer is an Analogy Based reasoner (Jehiel (2005)) which means that he has correct beliefs

[16]While we view a naive consumer as a consumer who intrinsically cannot evaluate products, because he lacks the expertise to do so, this is also consistent with an interpretation that naive consumers have prohibitively high costs to learn quality and product fit.

about the marginal distribution of qualities, match-values and prices in the market, but (possibly incorrectly) believes that they are independent from each other, thus failing to understand that price and quality might be correlated.[17] As a consequence, firms will be able to deceive naive consumers in the sense that their price choice does not betray their quality. In contrast, a fraction ν^S of "sophisticated" consumers, upon visiting a firm and examining its product, learns prices and product characteristics, and hence recognize deceptive products as what they are.

The sequence of events is as follows. At the outset, firms simultaneously, independently, and once and for all set q_k and p_k. Consumers then search (with perfect recall) until they purchase a product. That is, at any point in time, they decide whether to visit an additional firm at random at search costs $s > 0$, or to buy a product from a previously visited firm (and afterwards leave the market).[18]

Thus, a strategy for a firm is a quality-price combination, and a strategy for consumers is a search rule that specifies whether to end or continue search contingent on the past search history. We adopt a concept of equilibrium which is essentially Perfect Bayesian equilibrium except that naive consumers are Analogy Based reasoners. That is, we define an equilibrium as a strategy profile where firms and all consumers adopt optimal strategies given their beliefs about the other players' strategies. The beliefs of firms and sophisticated consumers are consistent with the other players' strategies, while naive consumers' beliefs are inconsistent in that they wrongly believe that a firm's quality and price choices are independent from each other. In effect, this means that although naive consumers are aware of firms that pursue a deceptive business model, they fail to identify such firms by evaluation of their products and pricing strategy.[19]

In the first step of our analysis, we establish necessary and sufficient conditions for the existence of an equilibrium which displays a segmentation of the market in a "candid" and a "deceptive" segment: a fraction λ of candid firms targets sophisticated consumers by offering quality $\overline{q}$ at a common price $\overline{p}$, and a fraction $1-\lambda$ of deceptive firms targets naive consumers by offering quality $\underline{q}$ at the price $\underline{p}$. Sophisticated consumers search until they find a suitable superior product, whereas naive consumers search until they find a sufficiently cheap (superior or deceptive)

[17]Our modeling choice is compatible with an interpretation that naive consumers are unaware of hidden product features and consequently make no (correct) inference by assumption. In that case, a naive consumer would believe that all firms in the market offer superior quality which, however, would not affect our results.

[18]We thus implicitly assume that consumers have no interest in leaving the market without purchasing a product.

[19]An equilibrium in our model thus formally corresponds to an Analogy Based equilibrium of an associated game in which each firm chooses its quality before its price and the analogy class of an Analogy Based reasoner is the set of all quality choices. In such an equilibrium all firms play the same mixed strategy (we thus do not"purify" the mixed equilibrium strategy) and an Analogy Based reasoner therefore believes that each firm's price and quality choice is independent of each other.

product.

Definition 1 *A triple* $(\lambda^*, \overline{p}^*, \underline{p}^*) \in (0,1) \times \mathbb{R}^2$ *is a segmented market equilibrium outcome if there is an equilibrium in which*

- *a fraction* λ^* *of firms offers a superior good with* $q_k = \overline{q}$ *and* $p_k = \overline{p}^*$*;*
- *a fraction* $1-\lambda^*$ *of firms offers a deceptive good with* $q_k = \underline{q}$ *and* $p_k = \underline{p}^*$*;*
- *sophisticated consumers never buy a deceptive good;*
- *naive consumers buy superior and deceptive goods.*

Note that the definition requires the share λ^* of candid firms to be interior. We refer to an equilibrium which supports a segmented market outcome as a segmented market equilibrium.

3 Conditions for market segmentation

In this section, we establish necessary and sufficient conditions for the existence of a segmented market equilibrium. We begin by deriving sophisticated consumers' optimal search rule in a segmented market equilibrium with outcome $(\lambda, \overline{p}, \underline{p})$.

Sophisticated consumer search

Because a searching consumer samples the next firm at random from the pool of all firms, a sophisticated consumer's optimal strategy is characterized by a reservation utility $\hat{U}^S$ which is the smallest level of utility that a consumer needs to obtain from purchasing a good so that he stops to search. As is shown in McCall (1970), the reservation utility is the current utility level that leaves a consumer indifferent between ending search in the current period and visiting a single additional firm. In a segmented market $(\lambda, \overline{p}, \underline{p})$, a sophisticated consumer expects to encounter a candid firm with probability λ and a deceptive firm with probability $1-\lambda$. Under the hypothesis that a sophisticated consumer does not buy a deceptive product, the reservation utility is therefore given recursively as

$$\hat{U}^S = \lambda \int_{\underline{\theta}}^{\overline{\theta}} \max\{\overline{q} + \theta - \overline{p}, \hat{U}^S\} \, dF(\theta) + (1-\lambda) \cdot \hat{U}^S - s. \tag{5}$$

It will often be more convenient to work with reservation match-values rather than reservation values. The reservation match-value is defined as the smallest match-value $\hat{\theta} \in \mathbb{R}$ of a superior product at which a sophisticated consumer stops and buys the product when the superior product is offered at the candid equilibrium price $\overline{p}$:

$$\overline{q} + \hat{\theta} - \overline{p} \equiv \hat{U}^S. \tag{6}$$

Define the function $g : \mathbb{R} \to \mathbb{R}$ by

$$g(z) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \max\{\theta - z, 0\}\, dF(\theta). \tag{7}$$

Then with (6), the consumer's reservation utility (5) can be written more succinctly as

$$g(\hat{\theta}) = \frac{s}{\lambda}. \tag{8}$$

As we show in Lemma A.1 in the appendix, the function g is strictly decreasing on $(-\infty, \overline{\theta}]$ with $g(-\infty) = \infty$ and $g(\overline{\theta}) = 0$. Therefore, equation (8) has a unique solution.

Finally, we verify when it is indeed optimal for a sophisticated consumer to refrain from a deceptive product for any match-value. This is the case if $\hat{U}^S \geq \underline{q} + \overline{\theta} - \underline{p}$, or, equivalently:

$$\overline{q} + \hat{\theta} - \overline{p} \geq \underline{q} + \overline{\theta} - \underline{p}. \tag{9}$$

Naive consumer search

Also the naive consumer's optimal search rule is characterized by a reservation utility. By McCall (1970), the reservation utility is equal to the expected utility the consumer believes to obtain in equilibrium. Under the hypothesis that a naive consumer buys superior and deceptive products, it is optimal for him to buy at the first firm. Because a naive consumer cannot observe the quality and the match-value of a product and fails to infer its quality from its price, he (wrongly) believes that any firm supplies him with the average valuation $\mathbb{E}(q + \theta) = \lambda\overline{q} + (1 - \lambda)\underline{q} + 0$. In contrast, he (correctly) believes that any firm charges $\overline{p}$ with probability λ and $\underline{p}$ with probability $1 - \lambda$. His reservation utility is therefore given as

$$\hat{U}^N = \lambda \cdot (\overline{q} - \overline{p}) + (1 - \lambda) \cdot (\underline{q} - \underline{p}) - s. \tag{10}$$

It will be useful to express a naive consumer's search rule in terms of a reservation price $\hat{p} \equiv \hat{U}^N - \mathbb{E}(q)$ which is the maximal price that he is effectively willing to pay for any product given his beliefs in a segmented market equilibrium. With (10), we obtain:

$$\hat{p} = \lambda \cdot \overline{p} + (1 - \lambda) \cdot \underline{p} + s. \tag{11}$$

Finally, we verify when it is indeed optimal for a naive consumer to buy superior and deceptive products. This is the case if $\underline{p} \leq \hat{p}$ and $\overline{p} \leq \hat{p}$, or, equivalently:[20]

$$\overline{p} - \underline{p} \leq \frac{s}{1 - \lambda} \quad \text{and} \quad \underline{p} - \overline{p} \leq \frac{s}{\lambda}. \tag{12}$$

[20]Note that although a naive consumer will purchase from the first firm he visits in equilibrium, he will continue search off the path when prices are large. A naive consumer in our setting is therefore different from a Varian (1980) type of "loyal" consumer who always purchases from the first firm he visits no matter what the price. That consumers do not "truly" search and do not visit more than one firm in equilibrium is common if consumers perceive products as homogeneous as, e.g., in Diamond (1971) and Stahl (1989).

Demand and firm profits

To determine firms' optimal pricing strategies, we now derive demand and profits in the two segments. The following lemma states the profits of a given firm k that charges price p_k taking as given that all other firms and consumers adopt the strategy as specified in a (candidate) equilibrium with outcome $(\lambda, \overline{p}, \underline{p})$. To state the lemma, we define for a given $\overline{p}$ and $\hat{\theta}$:

$$\Pi(p,q,c) = \nu^N \cdot (p-c) + \frac{\nu^S}{\lambda} \cdot \frac{1-F\left(\hat{\theta}+(p-\overline{p})-(q-\overline{q})\right)}{1-F(\hat{\theta})} \cdot (p-c). \tag{13}$$

Lemma 1 *In a segmented market equilibrium with outcome $(\lambda, \overline{p}, \underline{p})$, the profit of a firm that sets quality q_k and price p_k is given by*

$$\pi(q_k,p_k) = \begin{cases} \Pi(p_k,q_k,c(q_k)) & if \quad p_k \leq \hat{p}; \\ \Pi(p_k,q_k,c(q_k)) - \nu^N(p_k - c(q_k)) & if \quad p_k > \hat{p}, \end{cases} \tag{14}$$

where $\hat{p}$ is given by (11). Equilibrium profits are given by

$$\pi(\underline{q},\underline{p}) = \nu^N \cdot (\underline{p} - \underline{c}) \quad and \quad \pi(\overline{q},\overline{p}) = (\nu^N + \frac{\nu^S}{\lambda}) \cdot (\overline{p} - \overline{c}). \tag{15}$$

To derive some intuition for the lemma, note that the profit of a firm is given as the product of the mass of consumers who visit the firm in its lifetime, the probability that a visitor actually purchases the good, and the firm's mark–up $(p_k - c)$.

Now, in equilibrium, all naive consumers visit only one firm, because they buy in the first period and leave the market afterwards. Therefore, each firm attracts a mass ν^N of naive consumers in its lifetime. In contrast, all sophisticated consumers visit in expectation more than one firm, because they search until they find a satisfying, superior product. As is shown in the appendix, the mass of sophisticated consumers who visit the firm in its lifetime is $\nu^S/(\lambda(1-F(\hat{\theta})))$. Upon visiting, a sophisticated consumer actually purchases a firm's product if it supplies more utility than his reservation utility (6), whereas a visiting naive consumer buys the product if the firm charges a price below his reservation price (11), which is reflected in the case distinction in (14). From these observations, profits (14) can be derived straightforwardly.

To see what is behind the formula (15) for equilibrium profits, observe that the demand ν^N of naive consumers is split equally among the unit mass of all deceptive and candid firms. The demand ν^S of sophisticated consumers, in turn, is split equally among only the mass λ of candid firms. Expression (15) therefore simply reflects that a firm's profit is the product of its demand and its mark–up.

Equilibrium

Our next step is to characterize segmented market equilibrium outcomes formally. We restrict attention to equilibria with actual consumer search, that is, in which the equilibrium reservation match-value $\hat{\theta}^*$ is in the interior of the support of match-values. From a technical point of view, this facilitates tractability, as it allows us to characterize optimal firm behavior by first order conditions.[21] From now on, when we refer to "equilibrium", we mean such an equilibrium in which actual search takes place.

In the next lemma, we provide a system of equations whose solution corresponds to a segmented market equilibrium outcome. To ensure that first order conditions are also sufficient for optimality, we impose from now on the following regularity condition without further mention:

$$\frac{f'(\overline{\theta})}{f(\overline{\theta})} \geq -\frac{\nu^S}{\nu^N} \cdot f(\overline{\theta}). \tag{16}$$

Note that the condition always holds with increasing density.[22]

Lemma 2 $(\lambda^*, \overline{p}^*, \underline{p}^*)$ *is a segmented market equilibrium outcome if and only if there is* $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$*,* $\hat{p}^* \geq 0$*, and* $\lambda^* \in (0,1)$ *so that:*

$$\begin{aligned} g(\hat{\theta}^*) &= \frac{s}{\lambda^*}, & (17)\\ \hat{p}^* &= \lambda \cdot \overline{p}^* + (1-\lambda)\cdot \underline{p}^* + s, & (18)\\ \overline{p}^* - \overline{c} &= \frac{1+\lambda^* \frac{\nu^N}{\nu^S}}{h(\hat{\theta}^*)}, & (19)\\ \underline{p}^* &= \hat{p}^*, & (20)\\ \left(\nu^N + \frac{\nu^S}{\lambda^*}\right)\cdot(\overline{p}^* - \overline{c}) &= \nu^N \cdot (\underline{p}^* - \underline{c}). & (21)\end{aligned}$$

To understand Lemma 2, notice first that (17) restates sophisticated consumers' optimal search rule (8), and second that (18) restates naive consumers' optimal search rule (11). Third, (19)

[21]In principle, there may be equilibria with $\hat{\theta}^* = \underline{\theta}$ in which sophisticated consumers buy with probability 1 at the first candid firm they visit. Intuitively, if sophisticated strictly prefer to purchase at the first candid firm, then a candid firm's demand would be (locally) inelastic so that it would prefer to raise its price. In these equilibria, however, the fraction of candid firms adjusts such that sophisticated consumers are exactly indifferent between continuing and discontinuing search if they find a superior product which displays the worst match-value ($\hat{\theta}^* = \underline{\theta}$). A candid firm's profit function then has a kink at the equilibrium price so that its pricing behavior cannot be characterized by first order conditions.

[22]It is standard in the search literature to impose sufficient conditions similar to (16) which ensure that first order conditions are sufficient for profit maximization. In the absence of naive consumers ($\nu^N = 0$), our model is akin to Wolinsky (1986) who only requires that the hazard rate of F be increasing (see the footnote on page 504). This is consistent with (16), because if ν^N is sufficiently small, (16) is satisfied under the mild assumption $f(\overline{\theta}) > 0$, and because our assumption of log-concavity of f implies an increasing hazard rate. A condition similar to (16) is also adopted by Anderson and Renault (1999).

corresponds to the first order condition for $\overline{p}^*$ to be profit maximizing for a candid firm, expressed in terms of the candid mark–up $\overline{p}^* - \overline{c}$. Because of (16), the first order condition is also sufficient

Fourth, condition (20) says that $\hat{p}^*$ is the profit maximizing price for a deceptive firm. The reason is that, in equilibrium a deceptive firm only serves naive consumers. As a consequence, a deceptive firm's demand is inelastic up to the price $\hat{p}$, and it thus charges $\hat{p}$. In light of (18), this means that

$$\underline{p}^* = \overline{p}^* + \frac{s}{\lambda^*}. \tag{22}$$

Importantly, the mark–up of a deceptive firm is hence bounded from below by Δc, because $\overline{p}^* \geq \overline{c}$. Because a deceptive firm serves a mass ν^N of naive consumers in a segmented market equilibrium, this implies that a deceptive firm's profits (and hence equilibrium profits) are bounded from below by $\nu^N \dot{\Delta} c$.

Fifth, condition (21) says that candid and deceptive firms make the same profits. This has to hold in equilibrium, because otherwise firms in the less profitable segment would want to move to the more profitable one. Thus, the profit that a firm earns by selling deceptive products at a higher mark–up to only naive consumers is equal to the profit that a firm earns by selling superior products at a lower mark–up to naive and sophisticated consumers.

Observe that by (22) the deceptive price slightly exceeds the candid price. To understand this, note that the reservation price of a naive consumer strictly exceeds $\min\{\overline{p}, \underline{p}\}$ by (18). As a consequence, if the deceptive price was lower than the candid price, then a naive consumer would strictly prefer to purchase a deceptive product at $\underline{p}$. Then, a deceptive firm would want to raise its price, because it only serves naive consumers in a segmented market equilibrium. Therefore, the deceptive price must exceed the candid one. More intuitively, because naive consumers lack the expertise to evaluate products, they perceive products as homogeneous and hence have less incentives to search than sophisticated consumers which softens competition in the deceptive segment.

Equilibrium existence and uniqueness

We now ask when an equilibrium exists. Treating the share of candid firms λ as exogenous for the moment, the equilibrium conditions (17) and (19) pin down candid prices as function of λ as

$$\overline{p}^*(\lambda) - \overline{c} \equiv \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(g^{-1}(\frac{s}{\lambda}))}. \tag{23}$$

Inserting this, together with (22), in firm profits, we obtain the difference between a candid and a deceptive firm's profits as

$$\Delta(\lambda) \equiv \overline{\pi} - \underline{\pi} = \frac{\nu^S}{\lambda} \cdot \left(\overline{p}^*(\lambda) - \overline{c}\right) - \nu^N \cdot \left(\frac{s}{\lambda} + \Delta c\right). \tag{24}$$

The equal profit requirement (21) then amounts to the condition $\Delta(\lambda^*) = 0$.

The next proposition establishes that a unique segmented market equilibrium outcome always exists if search costs are sufficiently small.

Proposition 1 *For sufficiently small search costs, there is a unique segmented market equilibrium outcome. The equilibrium share of candid firms λ^* solves $\Delta(\lambda) = 0$, and equilibrium prices are pinned down by (20) and (23).*

The proof shows that, for sufficiently small search costs, the profit difference Δ is positive (resp. negative) if the share of candid firms is small (resp. large). An intermediate value argument then implies that $\Delta(\lambda) = 0$ has a solution in the range $(-\frac{s}{\underline{\theta}}, 1)$ which implies that the reservation match-value is interior: $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$.[23] To show uniqueness, it is then sufficient to argue that for sufficiently small search costs, the profit difference is decreasing at any equilibrium share of candid firms:

$$\frac{\partial \Delta}{\partial \lambda}(\lambda^*) < 0. \tag{25}$$

To understand more intuitively the role of search costs for the co-existence of candid and deceptive firms in equilibrium, suppose first that (almost) all firms are candid. Then, as search costs get close to zero, the mark–up for candid firms gets arbitrarily close to zero,[24] whereas the mark–up for deceptive firms is bounded from below by Δc. Therefore, if (almost) all firms are candid, there is a level of search costs below which candid firms earn lower profits than deceptive firms.

On the other hand, if very few firms are candid, then the demand for each candid firm is extremely large, because the demand from sophisticated consumers is divided up by very few firms. Thus, profits in the candid segment are very large. While the price of deceptive firms, $\underline{p} = \overline{p} + s/\lambda$, is also large if the candid segment is small, this price effect is diminished if search costs s are small, and therefore, if both s and λ are small, candid profits are higher than deceptive profits. As a consequence, there is an intermediate share of candid firms where the profits in the two segments equalize.

In what follows, we assume that search costs are such that a unique segmented market equilibrium exists. This allows us to study the comparative statics of segmented market equilibrium

[23]By Lemma A.1 in the appendix, $\hat{\theta}^*$ is in $(\underline{\theta}, \overline{\theta})$ if and only if $\lambda^* > -\frac{s}{\underline{\theta}}$. Because $\lambda^* \leq 1$, a segmented market equilibrium thus can only exist if search costs are sufficiently small and $-\frac{s}{\underline{\theta}} < 1$, which is a standard condition (for consumer search to take place) in the search literature (see e.g. Wolinsky (1986) and Anderson and Renault (1999)).

[24]Formally, as $s/\lambda \to 0$, equation (17) and the definition of g imply that $\hat{\theta}$ converges to $\overline{\theta}$. It follows that the mark–up $\overline{p} - c$ given by (19) converges to zero, because the hazard rate is unbounded.

outcomes. To do so, we will abuse notation and denote by $\overline{p}^*, \underline{p}^*$ and $\hat{\theta}^*$ both the equilibrium outcomes which depend only on exogenous parameters, as well as the "best–reply" functions $\overline{p}^*(\cdot), \underline{p}^*(\cdot)$ and $\hat{\theta}^*(\cdot)$ which are pinned down by (17-20) as functions of λ when treated as an independent variable such as in equation (23). In particular, we use total derivatives to indicate changes of equilibrium outcomes, and partial derivatives to indicate changes of the "best–reply", taking λ^* as given.[25]

4 Effects of stiffer competition

This section develops the main insights of our paper that stiffer competition in the form of lower search costs may have detrimental effects in our setting. We begin by showing that lower search costs may deteriorate the quality provision in the market, as it induces firms to engage in more deception. We then argue that this may have adverse welfare consequences. These results will form the basis of our policy implications.

Our first main result says that as search costs vanish, candid firms are entirely driven out of the market.

Proposition 2 *As search costs vanish, candid firms are entirely driven out of the market:* $\lim\limits_{s\to 0} \lambda^* = 0$.

In other words, when only some consumers are vulnerable to deception, then intense competition has the striking consequence that the market will supply only deceptive quality. In a nutshell, the intuition is that vanishing search costs allow sophisticated consumers to compare firms essentially for free. This intensifies competition in the candid segment, thus eliminating candid firms' mark–ups and profits, ultimately leading all firms to adopt the deceptive business model which guarantees positive profits.

This intuition hides the subtlety that what matters for sophisticated consumers is not nominal search costs, but the expected search costs to encounter a candid firm. If nominal search costs are one dollar, and among 100 firms there is only a single candid firm, a sophisticated consumer has to spend on average 100 dollars to find a candid firm. Thus, the *effective search costs* that a sophisticated consumer faces in equilibrium are the expected search costs

$$\sigma^* \equiv \frac{s}{\lambda^*} \tag{26}$$

[25] For example, $d\overline{p}^*/ds$ is the change of the candid equilibrium price with respect to s, whereas $\partial\overline{p}^*/\partial s$ is the change of the candid equilibrium price with respect to s when λ^* does not adjust to the change in s. Therefore: $d\overline{p}^*/ds = \partial\overline{p}^*/\partial s + \partial\overline{p}^*/\partial\lambda \cdot d\lambda^*/ds$.

to find a candid firm. A sophisticated consumer can inspect an additional candid firm essentially for free only if effective search costs vanish. In line with this, observe that candid mark–ups depend on search costs only through effective search costs:

$$\overline{p}^* - \overline{c} = \frac{1 + \lambda^* \frac{\nu^N}{\nu^S}}{h(g^{-1}(\sigma^*))}. \tag{27}$$

In what follows, we will thus interpret candid prices as a function of σ rather than s.

With this in mind, the intuition behind Proposition 2 can be made more precise. If the share of candid firms λ^* did not converge to zero, then effective search costs σ^* would tend to zero as s approaches zero, and candid firms' mark–ups would erode. Because firms make strictly positive profits in equilibrium, the erosion of mark–ups must be offset by an unbounded increase in the demand of a candid firm which, in turn, requires that the share of candid firms shrinks to zero (recall that in equilibrium a candid firm attracts the demand ν^S/λ^* from sophisticated consumers)—a contradiction.[26]

Proposition 2 shows that only the deceptive business model survives extremely stiff competition where search costs are virtually eliminated. This suggests that, at least when search costs are sufficiently small, lowering search costs induces firms to become deceptive. We now ask when it is indeed the case that more intense competition in the form of lower search costs reduces the share of candid firms in the market, and derive a necessary and sufficient condition for this to be the case. To state the condition, we introduce the elasticity of the candid mark–up with respect to effective search costs:[27,28]

$$\epsilon(\sigma) \equiv \frac{\partial(\overline{p}^* - \overline{c})/\partial\sigma}{(\overline{p}^* - \overline{c})/\sigma} = \frac{h'(\hat{\theta})}{h(\hat{\theta})} \cdot \frac{g(\hat{\theta})}{1 - F(\hat{\theta})}, \quad \text{with } \hat{\theta} = g^{-1}(\sigma). \tag{28}$$

We now show that the share of candid firms increases in search cost if and only if this elasticity is sufficiently large:

[26] Our model also displays the property that in the zero search cost limit, the market is frictionless in the sense that effective search costs vanish as search costs get small: $\lim_{s \to 0} \sigma^* = 0$. This also implies that the difference in prices between candid and deceptive firms, which equals σ^* by equation (22), vanishes as search costs go to zero. Formally, we establish that effective search costs vanish as auxiliary result (63) in the proof of Proposition 1.

[27] The calculation is provided in Lemma A.3 in the appendix.

[28] To understand why the elasticity of candid mark–ups depends on the steepness h' of the hazard rate, recall from (27) that $\overline{p}^*$ is inversely related to the hazard rate. This reflects that the hazard rate indicates the percentage increase in the mass of sophisticated consumers who will stop to search and buy the firm's product if it marginally decreases its price. Thus, the larger the hazard rate, the larger is a candid firm's demand elasticity, and the smaller is the equilibrium price. As a consequence, the price a candid firm charges increases sharply in σ if the hazard rate decreases sharply.

Proposition 3 *The share of candid firms increases in s if and only if*

$$\epsilon(\sigma^*) \geq \frac{\sigma^*}{\sigma^* + \Delta c}. \tag{C}$$

Before we explain the intuition behind the result, we shed light on when condition (C) is true. First, the condition is met if $\epsilon(\sigma^*) \geq 1$, that is, an increase in effective search costs of one percent results in an increase in a candid firm's mark–up that exceeds one percent. Moreover, it can be shown that condition (C) holds if (i) $f'(\hat{\theta}^*) \geq 0$, and if (ii) search costs are sufficiently small.[29] While (i) is stated in terms of the endogenous cutoff $\hat{\theta}^*$, we can turn it into primitive conditions by requiring it to hold for all θ. For example, (i) is satisfied for the uniform distribution. Finally, condition (C) is more likely to hold if Δc is large so that producing the deceptive quality entails large cost savings.

To see the intuition behind Proposition 3, note that the share of candid firms increases in search costs if, all else equal, candid profits increase by more than deceptive profits as search costs increase, because then more firms will adopt the candid business model in response. In order to illuminate the intuition behind Proposition 3, we therefore illustrate the economic forces that drive the evolution of profits in both market segments as search costs change.

Consider an increase of search costs by one dollar. This raises effective search costs by $1/\lambda^*$ dollars, all else equal. Because $\underline{p}^* = \overline{p}^* + \sigma^*$, a deceptive firm's price thus increases by $1/\lambda^*$ dollars more than the price of a candid firm. Now, deceptive and candid firms have the same number ν^N of naive customers, and a candid firm serves an additional ν^S/λ^* sophisticated customers. As search costs increase marginally, a deceptive firm thus earns $1/\lambda^*$ marginal dollars more than a candid firm per naive customer. Reversely, a candid firm earns $\partial \overline{p}^*/\partial \sigma \cdot 1/\lambda^*$ marginal dollars more from any of its sophisticated consumers. Hence, as search costs increase, the difference between the change in candid and deceptive profits is

$$\frac{\partial \Delta}{\partial s} = \frac{\partial}{\partial s}(\overline{\pi}^* - \underline{\pi}^*) = \frac{\nu^S}{\lambda^*} \cdot \frac{\partial \overline{p}^*}{\partial \sigma} \frac{1}{\lambda^*} - \frac{\nu^N}{\lambda^*}. \tag{29}$$

In what follows, we refer to $\partial \overline{p}^*/\partial \sigma^*$ as the "search effect". The search effect is unambiguously positive:[30] as effective search costs increase, sophisticated consumers become less picky in order to save on search costs, and the critical match-value $\hat{\theta}^*$ goes down. This renders the demand by sophisticated consumers less elastic (as the hazard rate is increasing) which allows candid firms to increase prices. The last term in (29), in turn, captures that (continued) search becomes more

[29]This insight is consistent with Proposition 2 that the share of candid firms actually vanishes as search costs vanish. A formal proof appears in Lemma A.4 in the appendix.

[30]This feature is analogous to the familiar property that in a search model without naive consumers such as Wolinsky (1986) or Anderson and Renault (1999), prices increase in search costs.

costly also for naive consumers, and as a consequence, deceptive firms can enforce even larger mark–ups against them.

The intuition behind Proposition 3 follows now from the observation that (29) is positive if the search effect is strong in the sense that $\partial \overline{p}^*/\partial \sigma$ is large. Then, as search costs increase, candid prices increase so sharply that candid profits grow faster than deceptive profits, and the share of candid firms consequently increases in s. Observe that the size of the search effect $\partial \overline{p}^*/\partial \sigma$ is proportional to the elasticity of candid mark–ups, as

$$\frac{\partial \overline{p}^*}{\partial \sigma} = \epsilon \cdot \frac{\overline{p}^* - \overline{c}}{\sigma^*}. \tag{30}$$

Condition (C) is now the necessary and sufficient condition for candid mark–ups, in equilibrium, to be sufficiently elastic so that the search effect is sufficiently strong.

If condition (C) is violated, then the share of candid firms decreases in s. Intuitively, if the candid mark–up elasticity is sufficiently small, then the search effect is so weak that (29) becomes negative. We now give a parameterized example in order to illustrate that this may indeed be the case. The example takes the exponential distribution, which displays a constant hazard rate, and adapts it to our setting with bounded support. (The calculations behind the example are provided in Appendix B.)

Example *Let* $[\underline{\theta}, \overline{\theta}] = [-1, 1]$, *and define*

$$f^E(\theta) = \begin{cases} e^{-(\theta+1)} & \theta < 0 \\ e^{-1} & \theta \geq 0. \end{cases} \tag{31}$$

Let $f = f^E$. *Then there is an open set of parameters* $s, \overline{q}, \underline{q}, \underline{c}, \overline{c}, \nu^N$ *so that a segmented market equilibrium exists with* $\hat{\theta}^* < 0$ *and*

$$\lambda^* = \frac{\frac{\nu^S}{\nu^N} - s}{\Delta c - 1}. \tag{32}$$

The example has the property that the equilibrium match-value $\hat{\theta}^*$ is below zero, and that, at $\hat{\theta}^*$, the distribution coincides with an exponential distribution. The hazard rate is thus constant and hence, the elasticity of candid mark–ups is zero, in violation of condition (C). As a result, the number of candid firms drops as s increases.

Welfare effects

We now argue that stiffer competition can be socially undesirable:

Proposition 4 *Suppose that condition (C) holds. Then total welfare increases in search costs if the efficiency gains from the superior quality,* $\Delta q - \Delta c$, *are sufficiently large.*

The basic intuition behind the result is that under condition (C) lower search frictions result in a larger share of deceptive products by Proposition 3. If deceptive products are very inefficient, this will outweigh the direct welfare costs associated with higher search costs.

To understand the forces behind Proposition 4 in more detail, the following lemma is key.

Lemma 3 *Equilibrium effective search costs are increasing in s: $d\sigma^*/ds \geq 0$.*

Therefore, effective and nominal search costs move in the same direction, and, as search frictions increase, searching for a suitable superior product becomes more tedious for sophisticated consumers, even if the share of candid firms increases in response. In this sense, effective search costs are well-behaved.[31]

Now, because prices only affect the division but not the size of total welfare, total welfare is given as the welfare of sophisticated and naive consumers as stated in (6) and (10) net of prices but accounting for the production costs of firms:[32]

$$W \quad = \quad \nu^S \cdot [\overline{q} - \overline{c} + \hat{\theta}^*] + \nu^N \cdot [\lambda^* \cdot (\overline{q} - \overline{c}) + (1 - \lambda^*) \cdot (\underline{q} - \underline{c}) - s]. \tag{33}$$

An increase in search costs affects welfare thus through three channels. First, effective search costs increase by Lemma 3 so that on average sophisticated consumers end up with a product with a lower match-value ($\hat{\theta}^*$ goes down). Second, the single search that naive consumers conduct gets more costly. These two effects resemble the welfare effects in standard search models and are unambiguously detrimental to total welfare. Third, however, search costs also affect the quality provision in the market. Under condition (C), the share of candid firms goes up with search costs so that naive consumers, on average, consume the inefficient deceptive product less often. If the associated efficiency gains are sufficiently large, then an increase in search costs improves total welfare.

Firm profits and consumer welfare

We now take a closer look at the division of total welfare and ask how the welfare gains from an increase in search costs are distributed among consumers and firms. To address this question, it

[31]To see the intuition behind the lemma, suppose to the contrary that effective search costs σ^* were decreasing in s. Hence, the share of candid firms λ^* would increase in s. By (27), this, first of all, would trigger an increase in candid prices, and second, because $\underline{p}^* = \overline{p}^* + \sigma^*$, the difference between deceptive and candid prices would shrink. For firms to nevertheless earn the same profit in the two segments after an increase in s, the demand that each candid firm receives from sophisticated consumers (ν^S/λ^*) would have to drop sharply. As it turns out, such a sharp increase of λ^* is impossible in equilibrium.

[32]Our model displays the property that a naive consumer's belief about his expected welfare is correct, although he wrongly expects to purchase superior products at $\underline{p}^*$ and deceptive products at $\overline{p}^*$. The reason is that he holds correct expectations about the average quality that he purchases and the average price that he pays.

is key to understand how equilibrium prices evolve as search costs change, since it is prices which determine the division of welfare. The next proposition states both the effect of higher search costs on equilibrium prices as well as on individual welfare.

Proposition 5 *Suppose condition (C) holds. Then we have:*

(o) Candid and deceptive prices increase in search costs.

(i) Firm profits increase in search costs.

(ii) Sophisticated consumer welfare decreases in search costs.

(iii) (True) naive consumer welfare increase in search costs if the difference in quality between superior and inferior products, Δq, is sufficiently large.

Parts (i)-(iii) follow straightforwardly from part (o): The profit of a deceptive firm (and hence of any firm) increases, because a deceptive firm's price increases by (o) and its demand remains unchanged (ν^N). Sophisticated consumer welfare (6) unambiguously drops, because a sophisticated consumer suffers from higher candid prices and larger effective search costs so that $\hat{\theta}^*$ goes down (as shown in (91)). A naive consumer benefits from the higher likelihood to encounter a candid firm, but suffers from higher market prices, the net effect being positive if the difference in quality between superior and inferior products, Δq, is sufficiently large.[33] Formally, naive consumer welfare (10) can be re-written with $\overline{p}^* = \underline{p}^* - s/\lambda^*$ as

$$\hat{U}^N \quad = \quad \underline{q} + \lambda^* \cdot \Delta q - \underline{p}^*. \tag{34}$$

Hence, $\hat{U}^N$ increases in s if Δq is sufficiently large, because λ^* increases in s by Proposition 3.

While part (o) resembles the price comparative statics in standard search models, we emphasize that condition (C) is critical for it to hold. In fact, we present a counter example below. To understand the intuition behind part (o), consider first candid prices. By (27), the effect of a change in search costs on candid prices is given as

$$\frac{d\overline{p}^*}{ds} = \frac{\partial \overline{p}^*}{\partial \sigma}\frac{d\sigma^*}{ds} + \frac{\partial \overline{p}^*}{\partial \lambda}\frac{d\lambda^*}{ds}. \tag{35}$$

Recall from (30) that the search effect $\partial p^*/\partial \sigma$ is positive. This, together with Lemma 3, immediately implies that the first term in (35) is positive. This captures that in response to an increase in effective search costs, sophisticated consumers become less picky in order to save on search costs, and visit and compare less firms in expectation, which, in turn, softens competition so that

[33] Note that (17-21) in Lemma 2 implies that the equilibrium market outcome is independent of Δq. Hence, Δq can be changed without affecting equilibrium variable such as λ^* and $\underline{p}^*$.

the candid price increases. The second term in (35) represents a novel effect which is due to the fact the share of candid firms is endogenous in our setting. In what follows, we refer to $\partial p^*/\partial\lambda$ as the "demand composition" effect. Recall that a candid firm's total demand is composed of the less elastic demand of naive consumers and the more elastic demand of sophisticated consumers. If the number of candid firms goes up, sophisticated consumers visit in expectation less firms until they find a satisfying superior product to purchase. As a consequence, there are in effect less sophisticated consumers present in the market and the share of naive consumers in a candid firm's demand increases. Because the demand from naive consumers is less elastic than the demand from sophisticated consumers, prices increase. The demand composition effect is thus positive: $\partial\overline{p}^*/\partial\lambda \geq 0$.[34]

We conclude that under condition (C), when the share of candid firms increases in s by Proposition 3, the second term on the right hand side of (35) is positive, and we obtain that candid equilibrium prices increase in s. Finally, note that because $\underline{p}^* = \overline{p}^* + \sigma^*$, Lemma 3 implies that if candid prices increase in s, so do deceptive prices.

We conclude this section with a remark on the price comparative statics when condition (C) is violated. In this case, the share of candid firms goes down in s, so that the second term on the right hand side of (35) is negative. We now argue that this may imply that candid prices actually decrease in search costs. As with Proposition 3, the size of the search effect depends on the steepness of the hazard rate and is zero in our example with (locally) constant hazard rate. Because in our example, the share of candid firms decreases in s, we thus infer that candid prices decrease in s.[35]

Example (ctd.) *Let $f = f^E$. Then there is an open set of parameters $s, \overline{q}, \underline{q}, \underline{c}, \overline{c}, \nu^N$ so that a segmented market equilibrium exists with $\hat{\theta}^* < 0$ and*

$$\overline{p}^* - \overline{c} = \frac{\Delta c - s\frac{\nu^N}{\nu^s}}{\Delta c - 1}. \tag{36}$$

Thus, candid prices are decreasing in search costs.

[34]Formally, by (27), we have $\partial\overline{p}^*/\partial\lambda = \nu^N/(\nu^S \cdot h(\hat{\theta}^*)) > 0$.

[35]When candid prices fall in search costs, the question arises whether this may lead to increasing sophisticated consumer welfare, or decreasing deceptive prices and profits. As it turns out, this is not the case even in the exponential example where the price decline of superior products induced by increasing search costs is extreme (because the hazard rate is constant). This suggests that in general, when this price decline is more moderate, welfare does not increase and profits do not decrease in search costs. We were, however, not able to verify this.

5 Effects of changes in naivete

This section addresses the question how (the division of) welfare depends on the number of naive consumers in the market. As outlined in the introduction, this question has received some attention in the literature, and is relevant to evaluate "transparency policies" that aim at fostering educating naive consumers.

Our next result confirms the intuition that as the number of naive consumers drops, the deceptive business model, which targets naive consumers, becomes less attractive so that the share of deceptive firms goes down. This is consistent with the existing literature which shows that the range of equilibria in which deception occurs expands in the number of naive consumers (Gabaix and Laibson (2006), Armstrong and Vickers (2012), Heidhues et al. (2017)). If the deceptive product is sufficiently inefficient, this will have the intuitive implication that welfare improves.

Proposition 6 *(i) The share of candid firms decreases in the fraction of naive consumers.*

(ii) Total welfare decreases in ν^N if

(a) a superior product is more efficient than a inferior product irrespective of match values: $\Delta q - \Delta c > \overline{\theta} - \underline{\theta}$, or

(b) the efficiency gains from purchasing a superior product exceed the expected search costs of finding one: $\Delta q - \Delta c > \sigma^*$.

While seemingly obvious, the complete argument for why an increase in the share of naive consumers results in a decrease in the share of candid firms is not entirely straightforward. The reason is that apart from a direct effect (more consumers are susceptible to deception), an indirect price effect works in the opposite direction. Precisely, as ν^N increases, a candid firm serves relatively more naive consumers whose demand is inelastic and all else equal the firm thus increases its price. This in turn implies that, all else equal, deceptive firms increase their prices by the same amount, because $\underline{p}^* = \overline{p}^* + \sigma^*$. However, as candid firms serve more consumers in equilibrium such an overall increase in prices benefits candid firms more than deceptive ones, and this effect makes offering superior products more attractive. As it turns out, the direct effect dominates the indirect effect, and overall, a firm's incentive to become deceptive indeed increases as ν^N increases.

As to part (ii), each naive and each sophisticated consumer generates less surplus. A naive consumer is simply more likely to end up with a deceptive product due to (i). A sophisticated consumer, on the other hand, faces larger effective search costs due to (i) and consequently buys a less suitable product on average ($\hat{\theta}^*$ goes down). Welfare thus decreases in ν^N if, in addition, the surplus an additional naive consumer generates is lower than that generated by the sophisticated consumer he replaces. The sufficient conditions stated in parts (a) and (b) of part (ii) ensure that this is the case.[36]

A prominent theme in the literature is how the welfare changes in response to a change in the number of naive consumers are distributed among sophisticated and naive consumers.[37] A novel negative spillover effect from naive to sophisticated consumers that arises in our framework is that an increase in the number of naive consumers increases the equilibrium effective search costs $\sigma^* = s/\lambda^*$ so that sophisticated consumers have to search longer to find a candid firm. This follows from part (i) of Proposition 6.

Clearly, consumer welfare does not only depend on effective search costs, but also on prices. As explained below, the direction in which equilibrium prices move as the number of naive consumers increases, is, in general, ambiguous. In the next proposition, however, we provide a sufficient condition so that both candid and deceptive prices increase in the number of naive consumers. This condition can be seen as a regularity condition which ensures that candid prices are well-behaved in the sense that candid prices drop when the share of candid firms (exogenously) increases. This, in turn, will imply that candid prices increase in the number of naive consumers and deliver clear-cut effects on individual welfare.

Proposition 7 *Let*

$$\epsilon(\sigma^*) \geq 1. \tag{37}$$

Then, as the share of naive consumers increases,

(i) candid and deceptive equilibrium prices increase;

(ii) firm profits increase;

(iii) sophisticated and naive consumer welfare decreases.

Part (ii) and (iii) follow straightforwardly from part (i) and our previous considerations: As ν^N increases, a deceptive (and hence a candid) firm's profit grows, because it sells deceptive products

[36]Notice that by a revealed preference argument, a sophisticated consumer obtains, in equilibrium, a larger utility than a naive consumer. This, however, does not imply that a sophisticated consumer generates more additional surplus, because a naive consumer generates a larger profit for firms.

[37]In Gabaix and Laibson (2006) and Armstrong and Vickers (2012), sophisticated consumers benefit from being cross-subsidized by naive ones, an effect that is absent in our model. In Heidhues et al. (2017), naive consumers harm sophisticated ones, because their presence pushes up prices and increases deception in the market. In Armstrong and Chen (2009), both naive and sophisticated consumer welfare might be non-monotone in the number of naive consumers.

at a higher price to a larger number of naive consumers. Sophisticated consumer welfare (6) goes down, because a sophisticated consumer suffers from higher candid prices by (i) and larger effective search costs as discussed earlier. Finally, naive consumer welfare (34) falls, because a naive consumer is more likely to end up with a deceptive product and has to pay more.

The more involved part is (i). By (27), the overall effect of an increase in the share of naive consumers on candid prices consists of various effects, and is formally given by

$$\frac{d\overline{p}^*}{d\nu^N} = \frac{\partial \overline{p}^*}{\partial \nu^N} + \frac{d\overline{p}^*}{d\lambda} \cdot \frac{d\lambda^*}{d\nu^N}. \tag{38}$$

To disentangle this expression, observe first that the direct effect $\partial \overline{p}^* / \partial \nu^N$ is unambiguously positive, because the demand of naive consumers is less elastic than the demand of sophisticated ones. Second, $d\lambda^*/d\nu^N < 0$ by Proposition 6. Therefore, (38) is positive if candid prices drop in response to a larger share of candid firms, that is, $d\overline{p}^*/d\lambda \leq 0$. Whereas the sign of $d\overline{p}^*/d\lambda$ is, in general, not clear-cut, the condition that $\epsilon(\sigma^*) \geq 1$ turns out to be a sufficient condition for $d\overline{p}^*/d\lambda \leq 0$.[38] Finally, observe that deceptive prices increase in ν^N if candid prices do, because $\underline{p}^* = \overline{p}^* + \sigma^*$ and σ^* increases in ν^N, since λ^* decreases in ν^N by Proposition 6.

6 Shrouding

While the previous results suggest that policies that educate naive consumers improve welfare, this section discusses the question whether firms themselves have incentives to educate naive consumers by unshrouding the hidden (bad) attributes of the deceptive products in the market. A key point in Gabaix and Laibson (2006) and Heidhues et al. (2017) is that equilibria can be sustained in which all firms are deceptive, even when a single deviating firm could educate all naive consumers to become sophisticated. The reason is that, given the equilibrium pricing of rival firms, a sophisticated consumers can only be attracted when charged below marginal cost. In our framework, if a single firm could unshroud and educate all naive consumers, then a candid firm would want to do so, because sophisticated consumers are profitable for candid firms. Hence, unshrouding increases the profit of each candid firm, because the demand of otherwise naive consumers is split equally only among candid firms.[39] In other words, candid firms steal the entire business from deceptive firms by unshrouding, benefiting each single candid firm.

[38] The underlying reason is that, analogously to the considerations in Proposition 5, an increase in the share of candid firms, first, reduces effective search costs, pushing prices down through the search effect, but, second, affects the composition of the demand of candid firms, as sophisticated consumers visit in expectation less firms, pushing prices up through the demand effect. The first effect dominates if ϵ is sufficiently large.

[39] In fact, this is a lower bound on the profits that a deviating candid firm could obtain, as it could also change its price in response to its decision to unshroud.

In our search framework, however, it seems natural to assume that a firm can only educate those consumers who actually visit the firm. As we argue next, in this case, no firm would educate visiting naive consumers. Suppose that any firm had the opportunity to unshroud so that together with its quality choice, it can also choose whether or not to turn any visiting naive consumer into a sophisticated one by educating him. We may think of two scenarios here. In the first scenario, the firm can only unshroud yet not adjust its price. While it is obvious that no deceptive firm has an interest to unshroud, this is also true for any candid firm,[40] because naive consumers, in contrast to sophisticated ones, purchase the firm's product in any case at the equilibrium price, that is, irrespective of product fit. A "business stealing effect" that could render unshrouding profitable is thus absent in our model, as unshrouding does not generate any additional demand for candid firms from otherwise naive consumers. This logic carries over to the second scenario in which the firm can unshroud and charge a price different from the equilibrium price. Intuitively, if a candid firm unshrouds so that all its customers are sophisticated, then the firm would optimally lower its price, as the demand from sophisticated consumers is more elastic than the demand from naive consumers. However, given a price lower than the equilibrium price, a candid firm cannot benefit from unshrouding, because at that price a naive consumer, in contrast to sophisticated consumer, would purchase in any case.

On a related note, recall that by Proposition 7 industry profits increase with the number of naive consumers so that reducing the number of naive consumers reduces profits in the "long-run".

7 Policy implications

In this section, we turn to the normative implications of our analysis and discuss policy interventions that may, or may not, improve (consumer) welfare in our framework.

Search costs

Our analysis implies that policy interventions that aim at lowering search costs may backfire. As shown in Proposition 4, the welfare benefits of markets with lower search frictions such as online markets are ambiguous, as they might increase the incentives for firms to become deceptive.

A key measure that aims at lowering search costs in practice is the standardization of price and product information so as to make it easier for consumer to evaluate prices and products. The harmonization of price information is a common regulatory tool in the EU for financial services

[40] Note that this implies that no deceptive firm has an interest in unshrouding and becoming candid at the same time.

(see the discussion in Piccione and Spiegler (2012)) or for grocery products (supermarkets are required to quote the price per 100g).

In practice, search frictions are also reduced by measures that promote e-commerce (such as lowering approval standards to register an online business). Moreover, certain information disclosure duties imposed on firms may reduce search costs if they improve consumers' access to product and price information.[41] Finally, search costs may also be reduced through measures that increase the number of firms in the market, e.g. by reducing the barriers of market entry, because this reduces the time to find (or the distance between) competitors.

Naivete

The welfare implications of policy interventions that aim at reducing the number of naive consumers are described in Proposition 6 and 7. Together with the result in the literature (see footnote 37), our results support a conclusion that such policy interventions are beneficial for total welfare, but there effects on the distribution of individual welfare are context-dependent.

Naivete can be reduced by information or education campaigns (e.g. fostering financial literacy in the context of banking services). If naivete is interpreted more broadly as a form of limited attention at the point of purchase, mandatory product labels which categorize products in eye-catching ways ("red", "yellow", "green") and trigger consumer awareness can be seen as a way to reduce naivete.[42]

Price regulation

At a more fundamental level, what ultimately causes welfare losses in our framework is that the presence of naive consumers guarantees positive profits from the deceptive business model which makes price competition dysfunctional. This raises the question whether regulating prices directly may be beneficial in our framework.[43]

To address this question, we consider a situation in which the regulator can impose a price floor $\overline{p}_F$, and we argue that a price floor exceeding the laissez-fare equilibrium price $\overline{p}^*$ can improve total and consumer welfare. To keep the analysis simple, we consider the effect of an exogenously imposed (marginal) increase of the candid equilibrium price from $\overline{p}^*$ to $\overline{p}_F = \overline{p}^* + dp$ for small $dp > 0$. If such an intervention is welfare improving, then it suggests that it is optimal to impose

[41] Information disclosure duties may, however, also make naive consumers aware of the deceptive nature of a product.

[42] Since boundedly rational consumers may have difficulties to process information correctly, such measures may, however, backfire. For example, categories such "green" or "red" may make consumers excessively negligent resp. diligent. Moreover, product labels are also likely to affect search costs. A comprehensive analysis of these issues requires a more detailed model of consumer naivete and is beyond the scope of this paper.

[43] We thank Tymon Tatur for raising this question.

a price floor above the equilibrium price.[44] By the equal profit requirement (24), the share of candid firms that balances the profits in the two segments given an arbitrary candid price $\overline{p}$ is

$$\lambda = \frac{\nu^S \cdot (\overline{p} - \overline{c}) - \nu^N \cdot s}{\nu^N \cdot \Delta c}. \tag{39}$$

The equilibrium effect of a price floor on the share of candid firms is then given by the effect of an increase of the equilibrium price $\overline{p}^*$ to $\overline{p}_F = \overline{p}^* + dp$. A price floor thus improves the quality provision in the market, as λ increases in $\overline{p}$ in (39). Intuitively, a candid firm has a larger demand than a deceptive firm so that as prices increase, the candid business model gets more profitable. This increase in the share of candid firms benefits both sophisticated and naive consumers, as the former face lower effective search costs and the latter are more likely to purchase a superior (more efficient) product. A price floor thus improves total welfare, because paid prices are merely a transfer from the point of view of total welfare. Moreover, if the difference between superior and inferior quality, Δq, is sufficiently large, then a price floor also improves total consumer welfare. The intuition is that if Δq is sufficiently large, then the gains of naive consumers (who are more likely to purchase a superior product) outweigh the losses (of all consumers) due to higher prices.

Lemma 4 *Imposing a price floor $\overline{p}_F = \overline{p}^* + dp$ always improves total welfare, and improves consumer welfare if the difference in quality between superior and inferior products, Δq, is sufficiently large.*

Minimum quality standard

If the regulator can observe quality directly, then imposing a minimum quality standard which requires firms to produce the superior product and thus effectively bans the deceptive product, would (clearly) improve total and consumer welfare. This is consistent with some regulatory policies that have been adopted in practice.[45]

However, in practice, a minimum quality standard may sometimes be only effective in preventing the most extreme forms of deception, because firms may, in response, design new deceptive products that satisfy the standard but still provide inefficiently low quality and allow firms to charge relatively high mark–ups to naive consumers. In a worst case scenario, a standard only

[44] It is best to consider our exercise as a first heuristic step of a more rigorous analysis that makes the (realistic) assumption that the regulator lacks relevant knowledge, e.g. about sellers' costs. The regulator then sets a price floor so as to maximize some objective, e.g. expected consumer welfare, anticipating the market outcomes that arises under a price floor regulation. Our analysis for a given cost realization suggests that an optimal price floor will be binding for low cost realizations.

[45] For instance, within the EU, consumers are entitled to reimbursement of add-on payments that are not explicitly agreed upon in the initial contract (see Article 22 of the Consumer Rights Directive 2011/83/EU). This can be interpreted as a ban of deception through add-on pricing. Likewise, after the financial crisis, financial supervisory bodies in the EU have been granted the authority to ban "complex" derivatives.

slightly improves the quality of a deceptive product, but does not improve the efficiency of a deceptive product at all. This raises the issue whether, in this case, a minimum quality standard is still welfare improving.

To explore this issue, suppose that a minimum quality standard $q_M \in (\underline{q}, \overline{q})$ with $dq = q_M - \underline{q} > 0$ is in place and firms can respond by offering either the superior product of quality $\overline{q}$ as before or a deceptive product of quality $q_M > \underline{q}$ which is equally inefficient than the "most" deceptive product that has been banned by the standard. That is, production costs for the deceptive quality q_M are $c_M = \underline{c} + (q_M - \underline{q})$. This guarantees that any welfare enhancing effects of a minimum quality standard do not simply stem for a higher efficiency of deceptive products. The next lemma shows that in this scenario, imposing a minimum quality standard, which is not too large, still improves total welfare and, depending on the mark–up elasticity with respect to search costs, may improve consumer welfare.

Lemma 5 *For sufficiently small $dq = q_M - \underline{q} > 0$, imposing a minimum quality standard $q_M = \underline{q} + dq$, with $c_M = \underline{c} + dq$, always improves total welfare, and improves consumer welfare if*

$$\epsilon(\sigma^*) \geq 1. \tag{40}$$

The intuition behind the lemma is straightforward. A minimum quality standard implies that a deceptive firm can save dq less on marginal production costs by producing the deceptive quality, but since it is still shunned by sophisticated consumers, it cannot attract additional demand through the higher quality. The deceptive business model thus becomes less attractive and the share of candid firms increases as a result of a minimum quality standard. Recall that this improves total welfare, because effective search costs drop and naive consumers are more likely to purchase the superior (more efficient) quality. Moreover, the same reasoning implies that consumer welfare increases if, in addition, candid and deceptive prices fall in response to a minimum quality standard. Now, if candid prices fall, then deceptive prices fall, because $\underline{p}^* = \overline{p}^* + s/\lambda^*$ and λ^* increases in response to a minimum quality standard as argued before. candid prices in turn fall if they are well-behaved, and the increase in the share of candid firms results in lower candid prices, that is, if $d\overline{p}^*/d\lambda < 0$ holds. As has been argued after Proposition 7, this is the case if $\epsilon(\sigma^*) \geq 1$ holds.

8 Discussion

In this section, we address the robustness of our findings to variations in how consumer naivete is modeled. First, we consider naive consumers who are merely uninformed but otherwise rational and make correct inferences from observed quality. Second, we consider the case that the naive consumers, like in our base model, fail to make correct inferences from observed prices but observe a noisy signal either about product quality or product fit. We will argue that in the first model variation, a segmented market equilibrium fails to exist, highlighting that our results are difficult to reconcile with rational consumer behavior. For the second model variation we show that our central result that superior products are driven out of the market if search frictions vanish still holds if the signal is sufficiently noisy. Therefore, the fact that naive consumer cannot assess products at all in our base model should be viewed as a simplifying assumption.

Rational but uninformed consumers

To study the role of naivete modeled as analogy based reasoning for the existence of a market segmentation equilibrium, suppose that instead of naive consumers there is a fraction of "uninformed" consumers who can neither observe product quality nor fit but are otherwise fully rational. Then, upon observing a firm's price, an uninformed consumer forms a belief $\hat{\lambda}(p)$ about the probability that the firm offers high quality. In (a Perfect Bayesian) equilibrium, this belief is consistent with the pricing strategies of firms, but can be arbitrary for prices "off the path" which are not chosen by firms in equilibrium. We impose the restriction that $\hat{\lambda}(p)$ is continuous so that a small change in a product's price does not result in a large change in a consumer's belief.[46] The following lemma shows that there exists no segmented market equilibrium.

Lemma 6 *Suppose there is a fraction of uninformed (but rational) consumers who hold continuous beliefs. Then there is no segmented market equilibrium.*

The intuition behind the lemma is that a separating equilibrium does not exist, because otherwise naive consumers would recognize deceptive products as what they are so that firms would have incentives to offer the efficient quality. On the other hand, a pooling equilibrium does not exist, because in such an equilibrium, uninformed consumers believe that all firms offer the same average quality at the same price. Hence, an uninformed consumer has strict incentives to buy at the current firm instead of continuing to search. As a consequence, because consumer beliefs are continuous, an uninformed consumer would still purchase from the current firm if it slightly increases its price. However, since a deceptive firm only serves uninformed consumers in a segmented market equilibrium, it would therefore indeed be profitable for the firm to slightly increase its price.

Noisy signals about product characteristics

To study the question how our results change when naive consumers have more information about product characteristics, we consider first the case that a naive consumer observes a noisy

[46] Absent any belief restriction, there exists a large number of pooling equilibria which rely on the fact that consumers believe that a firm offers inferior quality whenever it deviates from the equilibrium price.

signal about product quality if he visits a store but neither observes product fit nor makes correct inferences from observed prices. If naive consumer i inspects firm k, then he receives a signal $z_{ik} \in \{\underline{z}, \overline{z}\}$ with $\text{Prob}(\overline{z}|\overline{q}) = \text{Prob}(\underline{z}|\underline{q}) = 1/2 + \varepsilon$ where the parameter $\varepsilon \in [0, 1/2]$ measures the signal's precision. Signals are conditionally independent across consumers, conditional on the firm's quality. The following lemma shows that if ε is sufficiently small, then our central insight from Proposition 2 carries over, that is, as search costs vanish, candid firms are driven out of the market.

Lemma 7 *Suppose that naive consumers observe a noisy signal about product quality. Then, if the signal is sufficiently noisy (ε sufficiently small), candid firms are entirely driven out of the market as search costs vanish:* $\lim_{s \to 0} \lambda^* = 0$.

Intuitively, if the signal is too informative and (almost) fully reveals the product's quality to a naive consumer, then there does not exist an equilibrium in which firms offer the deceptive product. The reason is that otherwise there are efficiency gains from trading the efficient superior quality which could be split between the firm and its customers, because all consumers recognize products as what they are.[47] If the signal about product quality is, however, sufficiently noisy, then some firms offer the deceptive quality but lower their prices slightly (in comparison to the market outcome in our basic model) such that still all naive consumer, including those that receive a bad signal about the product's quality, purchase its product. This slightly reduces prices and profits in the deceptive segment, yet the equilibrium remains to be a segmented market equilibrium. Moreover, deceptive firms sell small volumes at a large mark–up whereas candid firms sell large volumes at small mark–ups. Loosely speaking, this implies that the intuition of Proposition 2 carries over: as search costs vanish, mark–ups in the candid segment erode and must be off-set by an unbounded increase in demand for candid firms, because the profits in the deceptive segment are bounded away from zero.

In a similar way, it can be shown that our main result carries over if naive consumers receive a sufficiently noisy signal about product fit. To see this, note first that Proposition 2 is driven by the fact mark–ups in the deceptive segment remain bounded away from zero as search costs vanish. This remains to be the case if the signal is sufficiently noisy, because a deceptive price must then still exceed the candid price. Otherwise, consider a naive consumer at a deceptive and (market-wide) cheapest firm who has observed the "worst" signal about product fit. The best this naive consumer can hope for by continuing search is to find, again, a cheapest (deceptive) firm, however, with the best signal about product fit. But if the signal is sufficiently noisy, the maximum gain in

[47]More precisely, the firm could offer the superior instead of the deceptive quality and raise its mark–up by an amount equal to the efficiency gains. As a naive consumers recognizes products as what they are, the firm still offers the same utility to the consumer (and hence looses no demand) but obtains a larger mark–up.

(expected) product fit is smaller than the costs to conduct this search. Therefore, the consumer would strictly prefer to buy at the current firm, which would induce it to raise its price. Thus, the cheapest price in the market cannot be charged by a deceptive firm, and the mark–up in the deceptive segment is still bounded away from zero as it exceeds Δc.

On a related note, if naive consumers can observe product fit sufficiently well, then two separate markets for naive and sophisticated consumers emerge: Naive consumer search for a suitable (cheap) deceptive product and ignore superior products, as they consider these products as too expensive, whereas sophisticated consumers search for a suitable (expensive) superior product and ignore deceptive products, as they recognize these products as what they are. In such a situation, the interaction between the two market segments, which is at the heart of our analysis, is rather limited, as the share of candid firms only influences the respective effective search costs in the two segments.

9 Conclusion

In this paper, we integrate consumer naivete in a search market model. The key idea is that the presence of naive consumers prevents competitive forces to unfold, because dumping inferior products on naive consumers is a safe profit haven. In fact, it is precisely in seemingly competitive markets with low search frictions where sellers have strong incentives to adopt the deceptive business model. Policy measures that aim at lowering search frictions should therefore be considered with caution.

A Appendix

Lemma A.1 *(i) g is strictly decreasing on $(-\infty, \overline{\theta})$ with $g(-\infty) = \infty$, $g(\underline{\theta}) = -\underline{\theta}$, and $g(\overline{\theta}) = 0$, and $g(z) = 0$ for all $z \geq \overline{\theta}$. Moreover, $g' = -(1-F)$.*
(ii) The inverse of g, denoted by g^{-1}, is well–defined and strictly decreasing on the domain $(0, \infty)$. By convention, we define $g^{-1}(0) = \overline{\theta}$. Note also that $g^{-1}(-\underline{\theta}) = \underline{\theta}$.

Proof of Lemma A.1 To see (i), note that for all $z < \overline{\theta}$, we have $g'(z) = -(1-F(z))$, so that g is strictly decreasing on $(-\infty, \overline{\theta})$. The values of g stated in the lemma follow straightforwardly by plugging in the respective arguments. Part (ii) is an immediate consequence of (i). ■

Proof of Lemma 1 The profit of a firm is given as the product of (a) the mass of consumers who visit the firm in its lifetime, (b) the probability that a visitor actually purchases the good, and (c) the firm's mark–up $(p_k - c)$.

As to (a), in a segmented market equilibrium, all naive consumers buy in the first period and leave the market afterwards. Thus, the mass of naive consumers who visit a given firm in its lifetime is equal to ν^N.

A sophisticated consumer leaves the market, in a segmented market equilibrium, if he is matched with a candid firm, and the match-value exceeds $\hat{\theta}$. This occurs with probability $\lambda \cdot (1 - F(\hat{\theta}))$. Because the mass of sophisticated consumers who visit a given firm in period t is equal to the mass of sophisticated consumers who have not left the market prior to t, this mass is given as $\nu^S \cdot [1 - \lambda(1 - F(\hat{\theta}))]^t$. Hence, the (expected) mass of sophisticated consumers who visit a given firm in its lifetime is equal to

$$\kappa = \nu^S \sum_{t=0}^{\infty} [1 - \lambda(1 - F(\hat{\theta}))]^t = \frac{\nu^S}{\lambda(1 - F(\hat{\theta}))}. \tag{41}$$

As to (b), a visiting sophisticated consumer i buys from firm k if $q_k + \theta_{ik} - p_k \geq \hat{U}^S$. By (5) and (6), this is the case if

$$q_k + \theta_{ik} - p_k \geq \overline{q} + \hat{\theta} - \overline{p} \quad \Leftrightarrow \quad \theta_{ik} \geq \hat{\theta} + (p_k - \overline{p}) - (q_k - \overline{q}). \tag{42}$$

Moreover, all visiting naive consumers buy as long as $p_k \leq \hat{p}$. A firm k that sets q_k and $p_k \leq \hat{p}$ therefore earns the profit

$$\pi(q_k, p_k) = \nu^N \cdot (p_k - c(q_k)) + \kappa \cdot \left[1 - F\left(\hat{\theta} + (p_k - \overline{p}) - (q_k - \overline{q})\right)\right] \cdot (p_k - c(q_k)). \tag{43}$$

Inserting (41) delivers (14). Finally, if a firm deviates to a price $p_k > \hat{p}$, it loses the profits $\nu^N(p_k - c(q_k))$ it would otherwise make from naive consumers.

The expressions for equilibrium profits in (15) follow immediately by inserting equilibrium prices $\overline{p}$ and $\underline{p}$ and the respective qualities $\overline{q}$ and $\underline{q}$ in (14), taking into account that $\underline{p} \leq \hat{p}$ in a segmented market equilibrium. ■

Lemma A.2 *Let $\hat{\theta}$ be given, and let*

$$\overline{p} = \frac{1+\lambda\frac{\nu^N}{\nu^S}}{h(\hat{\theta})} + c, \quad p_L = (\underline{\theta} - \hat{\theta}) + \overline{p}, \quad p_H = (\overline{\theta} - \hat{\theta}) + \overline{p}. \tag{44}$$

Then the function $\Pi(\cdot, \overline{q}, c)$ as defined in (13) is maximized on the domain (p_L, p_H) by $p^* = \overline{p}$.

Proof of Lemma A.2 Notice first that $p^* = \overline{p}$ is indeed in (p_L, p_H). The necessary first order condition for $p^* = \overline{p}$ to maximize $\Pi(\cdot, \overline{q}, c)$ is

$$0 = \frac{\partial \Pi(p^*, \overline{q}, c)}{\partial p} = \nu^N + \frac{\nu^S}{\lambda} - \frac{\nu^S}{\lambda} \cdot \frac{f\left(\hat{\theta}\right)}{1 - F(\hat{\theta})} \cdot (p^* - \overline{c}). \tag{45}$$

By (44), $p^* = \overline{p}$ therefore satisfies the first order condition.

To see that $p^* = \overline{p}$ is a global maximizer, we show that $\Pi(\cdot, \overline{q}, c)$ is quasi-concave on (p_L, p_H). In fact, we show the stronger property that $\Pi(\cdot, \overline{q}, c)$ is log-concave. To see this, let $\tau = \tau(p) = \hat{\theta} + (p - \overline{p})$ and define

$$D(p) \equiv \nu^N + \frac{\nu^S}{\lambda} \cdot \frac{1 - F(\tau)}{1 - F(\hat{\theta})}. \tag{46}$$

Observe that $\Pi(p, \overline{q}, c) = D(p) \cdot (p - c)$, and recall that the product of log-concave functions is log-concave. Because $(p - c)$ is log-concave in p, $\Pi(\cdot, \overline{q}, c)$ is therefore log-concave if D is log-concave. To show this, assume to the contrary that there is a p so that $(\log D)''(p) > 0$. As we show below,

$$(\log D)''(p) > 0 \quad \Rightarrow \quad (\log D)''(p_H) > 0. \tag{47}$$

The right inequality in turn is equivalent to[48]

$$D''(p_H) \cdot D(p_H) - D'(p_H)^2 > 0. \tag{48}$$

With $\alpha \equiv \frac{\nu^S}{\lambda(1-F(\hat{\theta}))}$, we have

$$D'(p) = -\alpha f(\tau), \quad \text{and} \quad D''(p) = -\alpha f'(\tau). \tag{49}$$

Because $D' < 0$ and $D > 0$, dividing (48) by $D'(p_H) \cdot D(p_H)$ and re-arranging terms yields

$$\frac{f'(\overline{\theta})}{f(\overline{\theta})} < -\frac{\alpha f(\overline{\theta})}{\nu^N + \alpha(1 - F(\overline{\theta}))} = -\frac{\alpha f(\overline{\theta})}{\nu^N} < -\frac{\nu^S}{\nu^N} f(\overline{\theta}), \tag{50}$$

[48]Note that $\frac{d^2}{dp_k^2} \log D = \frac{d}{dp_k}(D'/D) = 1/D^2 \cdot (D''D - D'^2)$.

where the final inequality follows from $\alpha > \nu^S$. But this inequality contradicts (16) which establishes that D is quasi-concave, and hence $\Pi(\cdot, \overline{q}, c)$ is log-concave as desired.

To complete the proof, we have to show (47). Notice that with (49) we have

$$(\log D)''(p) = \frac{D'(p)}{D(p)} \cdot \left[\frac{f'(\tau)}{f(\tau)} - \frac{D'(p)}{D(p)}\right], \tag{51}$$

and that $\frac{D'(p)}{D(p)}$ is negative by (49). Therefore, if the term in the square bracket is decreasing at p whenever $(\log D)''(p)$ is positive, then $(\log D)''(p)$ remains positive once it is positive and (47) follows. To see that the term in the square brackets is indeed decreasing at p, note that $(\log D)''(p) > 0$ implies that $(\log D)'(p) = \frac{D'(p)}{D(p)}$ is increasing at p, and our assumption that f is log-concave implies that $\frac{f'(\tau)}{f(\tau)}$ is decreasing at p. ■

Proof of Lemma 2 "$\Leftarrow$": Let $\theta^* \in (\underline{\theta}, \overline{\theta})$ and $\lambda^* \in (0, 1)$ be given and suppose (17–21) hold. We first show that consumer behavior is as required in a segmented market equilibrium. By (22), we have $\underline{p}^* = \overline{p}^* + s/\lambda^*$. Condition (12) is thus met. Therefore, naive consumers purchase both superior and deceptive products, and (18) re-states their optimal search rule (11). Also condition (9) is met, because $\underline{p}^* > \overline{p}^*$ by (22) and $\overline{\theta} - \hat{\theta} < \Delta q$ where the latter inequality follows from $\theta^* \in (\underline{\theta}, \overline{\theta})$ and assumption (3). Therefore, sophisticated consumer indeed prefer not to purchase deceptive products, and (17) re-states their optimal search rule (8).

Consequently, because consumer behavior conforms with a segmented market equilibrium, a firm's profit function is given by Lemma 1. What remains to be shown is that $(\overline{q}, \overline{p}^*)$ and $(\underline{q}, \underline{p}^*)$ with $\overline{p}^*$ and $\underline{p}^*$ as defined in (19) and (22) are indeed profit maximizing strategies for firms. Notice that by (21) firms earn equal profits in the two segments so that it suffices to show that $p_k = \overline{p}^*$ is profit maximizing given $\overline{q}$, and $p_k = \underline{p}^*$ is profit maximizing given $\underline{q}$, as in this case, (21) ensures that a deviation to the other segment is not profitable.

We begin with a candid firm with $q_k = \overline{q}$. By Lemma 1, we have that

$$\pi(\overline{q}, p_k) \leq \Pi(p_k, \overline{q}, \overline{c}) \quad \text{for all } p_k, \qquad \text{and} \qquad \pi(\overline{q}, \overline{p}^*) = \Pi(\overline{p}^*, \overline{q}, \overline{c}). \tag{52}$$

Hence, Lemma A.2 implies that $\overline{p}^*$ is profit maximizing within the price range $[p_L, p_H]$. Hence, it remains to show that a candid firm cannot improve by choosing a price outside of $[p_L, p_H]$.

Consider first a price $p_k < p_L$. At such a price, any sophisticated and naive consumer for any match-value realization strictly prefers to purchase at the current firm k rather than visit the next firm. Thus, the firm's demand would be locally inelastic, and it would be profitable for the firm to slightly increase the price. Hence, $p_k < p_L$ is not optimal.

Consider next a price $p_k > p_H$. At such a price any sophisticated consumer for any match-value realization strictly prefers to visit the next firm rather than purchase at the current firm k, and the firm at most sells to naive consumers. So suppose that a naive consumer indeed purchases a superior product (and hence also a deceptive product) at p_k such that the firm makes profits of $\nu^N \cdot (p_k - \overline{c})$. We infer:

$$\pi(\overline{q}, p_k) = \nu^N \cdot (p_k - \overline{c}) < \nu^N \cdot (p_k - \underline{c}) = \pi(\underline{q}, p_k) \leq \pi(\underline{q}, \underline{p}^*) = \pi(\overline{q}, \overline{p}^*), \tag{53}$$

where the second inequality follows from $\overline{c} > \underline{c}$, and the third equality follows because at p_k a deceptive firm only derives demand from naive consumers, as a sophisticated consumer, who does not purchase a superior product at p_k, neither purchases a deceptive product at p_k. The fourth inequality follows, as $\underline{p}^*$ is optimal given $\underline{q}$ (which we show next), and the final equality follows from (21). This proves that it is optimal for a candid firm to charge $\overline{p}^*$.

To see that it is optimal for a deceptive firm to charge $\underline{p}^*$, consider first a price $p_k > \underline{p}^*$. Because condition (9) is met in (the candidate) equilibrium, at $\underline{p}^*$ (and any larger price $p_k > \overline{p}^*$) a deceptive firm derives no demand from sophisticated consumers. Moreover, because $\underline{p}^* = \hat{p}$ by (18), we have $p_k > \hat{p}$, so that the firm neither derives demand from naive consumers. Consequently, a deviation to $p_k > \underline{p}^*$ is not profitable.

Next, consider $p_k < \underline{p}^*$. Observe that because the demand of naive consumers is inelastic up to the price $\hat{p}^*$ and $\underline{p}^* = \hat{p}^*$ by (18), setting a price $p_k < \underline{p}^*$ can only be profitable if it generates additional demand from sophisticated consumers, that is, if $p_k < p_H - \Delta q$.[49] Now, at very low prices $p_k < p_L - \Delta q$, any visiting naive and sophisticated consumer would purchase the firm's product so that the firm's demand would be perfectly inelastic. Accordingly, it would be profitable to raise prices. Hence, $p_k < p_L - \Delta q$ is not optimal.

Thus, it remains to consider prices $p_k \in [p_L - \Delta q, p_H - \Delta q]$. Observe that as $p_k < \hat{p}^*$, it holds that

$$\pi(\underline{q}, p_k) = \Pi(p_k, \underline{q}, \underline{c}) \leq \Pi(p_k, \underline{q}, \overline{c} - \Delta q), \tag{54}$$

where the second inequality follows, because $\Pi(p, q, c)$ is decreasing in c and because $\Delta q > \Delta c$ by (3). From definition (13), we infer that

$$\Pi(p_k, \overline{q}, \overline{c} - \Delta q) = \Pi(p_k + \Delta q, \overline{q}, \overline{c}), \tag{55}$$

and note that $p_k + \Delta q \in [p_L, p_H]$, because $p_k \in [p_L - \Delta q, p_H - \Delta q]$. By Lemma A.2, we thus have

$$\Pi(p_k + \Delta q, \overline{q}, \overline{c}) \leq \Pi(\overline{p}^*, \overline{q}, \overline{c}). \tag{56}$$

With (52) and (21), we can conclude that

$$\pi(\underline{q}, p_k) \leq \Pi(\overline{p}^*, \overline{q}, \overline{c}) = \pi(\overline{q}, \overline{p}^*) = \pi(\underline{q}, \underline{p}^*). \tag{57}$$

[49] By definition, p_H is the largest price that a sophisticated is willing to pay for a superior product that displays the best match-value $\overline{\theta}$. Therefore, at $p_k \geq p_H - \Delta q$, a deceptive firm derives no demand from sophisticated consumers.

Hence, a deviation from $\underline{p}^*$ to $p_k \in [p_L - \Delta q, p_H - \Delta q]$ is not profitable for a deceptive firm. This completes the proof of the "$\Leftarrow$"- part.

"$\Rightarrow$ ": If there is a segmented market equilibrium, then there must exist $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ and $\lambda^* \in (0,1)$ by definition and, a sophisticated consumer's optimal search rule satisfies (8) which is the same condition as (17). A naive consumer's optimal search rule in turn satisfies (11) which is the same condition as (18). In a segmented market equilibrium, a deceptive firm only derives inelastic demand from naive consumers so that it charges the largest price such that naive consumers yet purchase its product. We hence have $\underline{p}^* = \hat{p}^*$ which is the same condition as (20). From (18) and (20) we conclude that $\overline{p}^* < \hat{p}^*$ in any segmented market equilibrium. Because Lemma 1 applies, a candid firm's profits are thus given by $\pi(\overline{q}, p) = \Pi(p, \overline{q}, \overline{c})$ for p from an open interval around $\overline{p}^*$, (as $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ and $\overline{p}^* < \hat{p}$). Hence, $\pi(\overline{q}, p)$ is differentiable at $\overline{p}^*$, and the corresponding first order condition for $\overline{p}^*$ to be profit maximizing, stated in (45), implies (19). Finally, candid and deceptive firms must make equal profits, as otherwise, firms in the less profitable segment would move to the other segment, and thus, (21) follows from (15) in Lemma 1. ■

Proof of Proposition 1 The following two claims imply the desired result.

Claim (i): There is a (unique) segmented market equilibrium outcome $(\lambda^*, \overline{p}^*, \underline{p}^*)$ *if and only if* $\Delta(\lambda) = 0$ *has a (unique) solution* λ^* *with* $-\frac{s}{\underline{\theta}} < \lambda^* < 1$.

To see Claim (i), observe that by Lemma 2, it is sufficient to show that (17-21) has a solution $(\hat{\theta}^*, \lambda^*, \hat{p}^*, \overline{p}^*, \underline{p}^*)$ with $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$, $\hat{p}^* \geq 0$ and $\lambda^* \in (0,1)$ if and only if $\Delta(\lambda^*) = 0$ and $\lambda^* > -\frac{s}{\underline{\theta}}$.

Indeed, it follows by Lemma A.1 that $\hat{\theta}^* = g^{-1}(\frac{s}{\lambda})$ is a solution to (17) with $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ if and only if $\lambda^* > -\frac{s}{\underline{\theta}}$. Note further that with (17), (19) is equivalent to

$$\overline{p}^* - \overline{c} = \frac{1 + \lambda \frac{\gamma^N}{\gamma^S}}{h(g^{-1}(\frac{s}{\lambda^*}))}. \tag{58}$$

Inserting this and (22) into (21) and re–arranging terms yields that λ^* is a solution to

$$\gamma^N \cdot \left(\frac{s}{\lambda} + \Delta c\right) = \frac{\gamma^S}{\lambda} \cdot \frac{1 + \lambda \frac{\gamma^N}{\gamma^S}}{h(g^{-1}(\frac{s}{\lambda}))}, \tag{59}$$

which is equivalent to $\Delta(\lambda^*) = 0$, and this completes the proof of claim (i).

Claim (ii): If search costs are sufficiently small, then $\Delta(\lambda) = 0$ *has a unique solution* λ^* *with* $-\frac{s}{\underline{\theta}} < \lambda^* < 1$.

To prove Claim (ii), we first show that a solution exists. Indeed, an intermediate value argument implies that $\Delta(\lambda) = 0$ has a solution in the range $(-\frac{s}{\underline{\theta}}, 1)$ if

$$\Delta\left(-\frac{s}{\underline{\theta}}\right) > 0 \quad \text{and} \quad \Delta(1) < 0. \tag{60}$$

We show that (60) is satisfied if s is sufficiently small.

To see that the left inequality in (60) is met for small s, insert $\lambda = -\frac{s}{\underline{\theta}}$ into Δ to obtain

$$\Delta\left(-\frac{s}{\underline{\theta}}\right) = \left(-\frac{\nu^S \underline{\theta}}{s} + \nu^N\right) \cdot \frac{1}{h(g^{-1}(-\underline{\theta}))} - \nu^N(-\underline{\theta} + \Delta c). \tag{61}$$

Because $\underline{\theta} < 0$, this expression converges to $+\infty$ as $s \to 0$, as desired.

To see that the right inequality in (60) is met for small s, insert $\lambda = 1$ into Δ to obtain

$$\Delta(1) = \frac{1}{h(g^{-1}(s))} - \nu^N(s + \Delta c). \tag{62}$$

Recall from (7) that $g(\overline{\theta}) = 0$. Therefore, $\lim_{s\to 0} g^{-1}(s) = \overline{\theta}$, and hence, because $\lim_{\theta \to \overline{\theta}} h(\theta) = \infty$ by assumption, we obtain that $\lim_{s\to 0} \Delta(1) = -\nu^N \Delta c < 0$, and this establishes (60).

To show that the solution to $\Delta(\lambda) = 0$ is unique for small s, we show first the auxiliary claim

$$\lim_{s\to 0} \frac{s}{\lambda^*(s)} = 0 \quad \text{for any solution } \lambda^* = \lambda^*(s) \text{ with } \Delta(\lambda^*) = 0. \tag{63}$$

It is clearly sufficient to show this for $\underline{\lambda}^* = \min\{\lambda^* | \Delta(\lambda^*) = 0\}$ (which exists by the continuity of Δ). Indeed, suppose to the contrary that there is a (sub)sequence s_n, $n = 1, 2, \ldots$ with $s_n \to 0$ so that the sequence $s_n/\underline{\lambda}^*(s_n)$ is bounded from below by some $\zeta > 0$, that is, $s_n/\underline{\lambda}^*(s_n) > \zeta$ for all n. This first of all implies that $\lim_{n\to\infty} \underline{\lambda}^*(s_n) = 0$. Moreover, it implies that there is some $\theta_0 < \overline{\theta}$ so that $g^{-1}(\frac{s_n}{\underline{\lambda}^*(s_n)}) < \theta_0$ for all n by Lemma A.1. Together with (24), it thus follows that

$$\lim_{n\to\infty} \Delta(\lambda^*(s_n), s_n) \geq \lim_{n\to\infty} \left\{ \left(\frac{\nu^S}{\lambda^*(s_n)} + \nu^N\right) \cdot \frac{1}{h(\theta_0)} - \nu^N\left(\frac{s_n}{\lambda^*(s_n)} + \Delta c\right)\right\} = \infty, \tag{64}$$

a contradiction to assumption that $\Delta(\lambda^*(s_n), s_n) = 0$ for all n, and this establishes (63).

To complete the proof of uniqueness, we show

$$\frac{\partial \Delta(\lambda^*)}{\partial \lambda} < 0 \quad \text{for any solution } \lambda^* \text{ with } \Delta(\lambda^*) = 0 \text{ and } s \text{ sufficiently small.} \tag{65}$$

This is sufficient for uniqueness by (60). Indeed, to compute $\partial \Delta / \partial \lambda$, observe first that by (19)

$$\frac{\partial}{\partial \lambda}(\overline{p}^* - \overline{c}) = \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} - \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) \cdot \frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)^2} \cdot \frac{\partial \hat{\theta}^*}{\partial \lambda} \tag{66}$$

$$= \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} - \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) \cdot \frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)^2} \cdot \frac{1}{1 - F(\hat{\theta}^*)} \cdot \frac{s}{(\lambda^*)^2}, \tag{67}$$

where in the second line, we have used first that the equilibrium property $g(\hat{\theta}^*) = s/\lambda^*$ implies $\partial \hat{\theta}^* / \partial \lambda^* = -1/g'(\hat{\theta}^*) \cdot (s/(\lambda^*)^2)$, and second that $g'(\hat{\theta}^*) = -(1 - F(\hat{\theta}^*))$ by Lemma A.1.

Because the hazard rate is increasing, (67) implies $\frac{\partial}{\partial\lambda}(\overline{p}^* - \overline{c}) \leq \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)}$, and hence

$$\begin{aligned}
\frac{\partial \Delta(\lambda^*)}{\partial \lambda} &= \left(-\frac{\nu^S}{(\lambda^*)^2}\right) \cdot (\overline{p}^* - \overline{c}) + \frac{\nu^S}{\lambda^*} \cdot \frac{\partial}{\partial \lambda}(\overline{p}^* - \overline{c}) + \nu^N \cdot \frac{s}{(\lambda^*)^2} && (68)\\
&\leq \left(-\frac{\nu^S}{(\lambda^*)^2}\right) \cdot (\overline{p}^* - \overline{c}) + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)} + \nu^N \cdot \frac{s}{(\lambda^*)^2} && (69)\\
&= -\frac{1}{\lambda^*}\Delta(\lambda^*) - \frac{1}{\lambda^*}\Delta c + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)} && (70)\\
&= -\frac{1}{\lambda^*}\Delta c + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)}, && (71)
\end{aligned}$$

where in the last line, we have used that $\Delta(\lambda^*) = 0$. Now observe that (63) implies that $\hat{\theta}^* = g^{-1}(s/\lambda^*) \to \overline{\theta}$ by Lemma A.1. Since the hazard rate is unbounded, $\lim_{\theta \to \overline{\theta}} h(\theta) = \infty$, and expression (71) becomes negative for sufficiently small s. This establishes (65) and completes the proof. ■

Lemma A.3 *We have*

$$\frac{\partial(\overline{p}^*(\sigma) - \overline{c})/\partial\sigma}{(\overline{p}^*(\sigma) - \overline{c})/\sigma} = \frac{h'(\hat{\theta})}{h(\hat{\theta})} \cdot \frac{g(\hat{\theta})}{1 - F(\hat{\theta})}, \quad \textit{with } \hat{\theta} = g^{-1}(\sigma). \tag{72}$$

Proof of Lemma A.3 Differentiating $\overline{p}^*$ as given in (27) with respect to σ and inserting σ^* yields

$$\frac{\frac{\partial}{\partial\sigma}(\overline{p}^* - \overline{c})}{\overline{p}^* - \overline{c}} = -\frac{h'(g^{-1}(\sigma^*))}{h(g^{-1}(\sigma^*))} \cdot \frac{\partial g^{-1}(\sigma^*)}{\partial \sigma}. \tag{73}$$

Observe that $\partial g^{-1}(\sigma^*)/\partial\sigma = 1/g'(g^{-1}(\sigma^*))$. Together with $g' = -(1 - F)$ by Lemma A.1, we obtain:

$$\frac{\frac{\partial}{\partial\sigma}(\overline{p}^* - \overline{c})}{\overline{p}^* - \overline{c}} \cdot \sigma^* = \frac{h'(g^{-1}(\sigma^*))}{h(g^{-1}(\sigma^*))} \cdot \frac{\sigma^*}{1 - F(g^{-1}(\sigma^*))} \tag{74}$$

The desired result follows then from $\hat{\theta}^* = g^{-1}(\sigma^*)$ by (17). ■

Proof of Proposition 2 Contrary to the claim suppose that there is a (sub)sequence s_n, $n = 1, 2, \ldots$ with $s_n \to 0$ so that $\lambda^*(s_n)$ is bounded from below by some $\underline{\lambda}$, that is, $\lambda^*(s_n) > \underline{\lambda}$ for all n. Then, $\lim_{n\to\infty} s_n/\lambda^*(s_n) = 0$ holds, and Lemma A.1 implies that $\lim_{n\to\infty} g^{-1}(\frac{s_n}{\lambda^*(s_n)}) = \overline{\theta}$. We can then infer that

$$\lim_{n\to\infty} \Delta(\lambda^*(s_n), s_n) < \left(\frac{\nu^S}{\underline{\lambda}} + \nu^N\right) \cdot \lim_{\theta \to \overline{\theta}} \frac{1}{h(\theta)} - \nu^N \Delta c < 0, \tag{75}$$

where the last inequality follows from the fact that the hazard rate diverges as θ approaches $\overline{\theta}$. This contradicts the equilibrium condition that $\Delta(\lambda^*(s_n), s_n) = 0$ for all n. ■

Proof of Proposition 3 By the equilibrium property $\Delta(\lambda^*, s) = 0$, we have that $d\lambda^*/ds = -(\partial\Delta/\partial s)/(\partial\Delta/\partial\lambda)$. Because $\partial\Delta(\lambda^*)/\partial\lambda < 0$ by (65), it suffices to show that

$$\frac{\partial\Delta(\lambda^*)}{\partial s} \geq 0 \quad \Leftrightarrow \quad \epsilon(\sigma^*) \geq \frac{\sigma^*}{\sigma^* + \Delta c}. \tag{76}$$

As is argued in the text, differentiating (24) with respect to s yields (29):

$$\frac{\partial\Delta(\lambda^*)}{\partial s} = \frac{\nu^S}{\lambda^*} \cdot \frac{\partial\overline{p}^*}{\partial\sigma}\frac{1}{\lambda^*} - \frac{\nu^N}{\lambda^*}, \tag{77}$$

which by the definition of ϵ in (28) is equivalent to

$$\frac{\partial\Delta(\lambda^*)}{\partial s} = \frac{\nu^S}{\lambda^*} \cdot (\overline{p}^* - \overline{c}) \cdot \epsilon(\sigma^*) \cdot \frac{1}{s} - \frac{\nu^N}{\lambda^*}. \tag{78}$$

By (24), $\Delta(\lambda^*) = 0$ is equivalent to

$$\frac{\nu^S}{\lambda^*} \cdot (\overline{p}^* - \overline{c}) = \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c\right), \tag{79}$$

and (78) becomes

$$\frac{\partial\Delta(\lambda^*)}{\partial s} = \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c\right) \cdot \epsilon(\sigma^*) \cdot \frac{1}{s} - \frac{\nu^N}{\lambda^*} = \frac{\nu^N}{\lambda^*} \cdot \left[\left(\frac{s + \lambda^*\Delta c}{s}\right) \cdot \epsilon(\sigma^*) - 1\right]. \tag{80}$$

Finally, (80) is equivalent to (76), which completes the proof. ■

Lemma A.4 *Condition (C) holds if:*
(i) $f'(\hat{\theta}^*) \geq 0$, *or*
(ii) search costs are sufficiently small.

Proof of Lemma A.4 By (76), it suffices to show that $\partial\Delta(\lambda^*)/\partial s \geq 0$ if (i) $f'(\hat{\theta}^*) \geq 0$, or (ii) search costs are sufficiently small.

As to (i), notice first that a straightforward calculation yields that

$$\frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)} = \frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)} + h(\hat{\theta}^*) \tag{81}$$

and $\epsilon(\sigma^*)$ in (28) becomes

$$\epsilon(\sigma^*) = \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)} + h(\hat{\theta}^*)\right) \cdot \frac{g(\hat{\theta}^*)}{1 - F(\hat{\theta}^*)} \tag{82}$$

with $\hat{\theta}^* = g^{-1}(\sigma^*)$. Inserting this and $\overline{p}^*$ from (27) in (78) yields

$$\frac{\partial\Delta(\lambda^*)}{\partial s} = \left(\frac{\nu^S}{\lambda^*} + \nu^N\right)\left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)h(\hat{\theta}^*)} + 1\right)\frac{1}{\lambda^* \cdot (1 - F(\hat{\theta}^*))} - \frac{\nu^N}{\lambda^*}. \tag{83}$$

Observe that, as desired, this expression is positive if $f'(\hat{\theta}^*) \geq 0$, because $\lambda^* \cdot (1 - F(\hat{\theta}^*)) < 1$.

As to (ii), note that it suffices to show that (83) is positive as search costs get small. Recall that by Footnote 26, effective search costs σ^* vanish as $s \to 0$, so that $\hat{\theta}^* \to \overline{\theta}$ as $s \to 0$ by Lemma A.1. We hence find that

$$\lim_{s \to 0} \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*) h(\hat{\theta}^*)} + 1 \right) \frac{1}{(1 - F(\hat{\theta}^*))} = \lim_{\hat{\theta}^* \to \overline{\theta}} \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*) h(\hat{\theta}^*)} + 1 \right) \frac{1}{(1 - F(\hat{\theta}^*))}. \tag{84}$$

Notice that (83) is positive as search costs get small if (84) diverges. The reason why (84) diverges is that, on the one hand, $f'(\hat{\theta}^*)/f(\hat{\theta}^*)$ is bounded from below,[50] while the hazard rate diverges by assumption, and on the other hand, $1 - F(\hat{\theta}^*)$ tends to zero as $\hat{\theta}^* \to \overline{\theta}$. ■

Proof of Lemma 3 We have to show that

$$\frac{d\sigma^*}{ds} \geq 0. \tag{85}$$

Recall that $\sigma = s/\lambda$, and define the function

$$\tilde{\Delta}(\sigma) = \Delta\left(\frac{s}{\sigma}\right) = \left(\nu^S \cdot \frac{\sigma}{s} + \nu^N\right) \cdot \frac{1}{h(g^{-1}(\sigma))} - \nu^N(\sigma + \Delta c). \tag{86}$$

The equilibrium condition $\Delta(\lambda^*) = 0$ becomes $\tilde{\Delta}(\sigma^*) = 0$, and hence

$$\frac{d\sigma^*}{ds} = -\frac{\partial \tilde{\Delta}/\partial s}{\partial \tilde{\Delta}/\partial \sigma}. \tag{87}$$

Differentiating (86) yields that

$$\frac{\partial \tilde{\Delta}(\sigma^*)}{\partial \sigma} = \frac{\partial \Delta(\lambda^*)}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma} = \frac{\partial \Delta(\lambda^*)}{\partial \lambda} \left(-\frac{s}{\sigma^*}\right), \tag{88}$$

and $\partial \tilde{\Delta}(\sigma)/\partial \sigma$ is thus positive, because $\partial \Delta(\lambda^*)/\partial \lambda < 0$ by (65). Moreover, by (86):

$$\frac{\partial \tilde{\Delta}}{\partial s} = -\nu^S \frac{\sigma^*}{s^2} \cdot \frac{1}{h(g^{-1}(\sigma^*))}. \tag{89}$$

Because this is negative, it follows that $d\sigma^*/ds$ is positive, as desired. ■

Proof of Proposition 4 By (33),

$$\frac{dW}{ds} = \nu^S \cdot \frac{d\hat{\theta}^*}{ds} + \nu^N \cdot \frac{d\lambda^*}{ds} \cdot (\Delta q - \Delta c) - \nu^N. \tag{90}$$

By (17), we have $g(\hat{\theta}^*) = \sigma^*$, and hence

$$\frac{d\hat{\theta}^*}{ds} = \frac{d\sigma^*}{ds} \frac{1}{g'(\hat{\theta}^*)}, \tag{91}$$

[50] By log-concavity of f, f'/f is decreasing and $f'(\overline{\theta})/f(\overline{\theta})$ is bounded from below by (16) and $f(\overline{\theta}) > 0$.

which is negative, because $g' = -(1-F)$ by Lemma A.1 and $d\sigma^*/ds \geq 0$ by Lemma 3. However, if Condition (C) holds, then $d\lambda^*/ds > 0$ by Proposition 2, and (90) is positive if $\Delta q - \Delta c \geq 0$ is sufficiently large. ■

Proof of Proposition 5 The argument is given in the main text. ■

Proof of Proposition 6 As to (i). Because $d\lambda^*/d\nu^N = -(\partial\Delta/\partial\nu^N)/(\partial\Delta/\partial\lambda)$ and $\partial\Delta/\partial\lambda < 0$ by (65), it is sufficient to show that

$$\frac{\partial\Delta}{\partial\nu^N} < 0. \tag{92}$$

Recall that $\Delta = (\nu^N + \nu^S/\lambda^*)\cdot(\overline{p}^* - \overline{c}) - \nu^N\cdot(\overline{p}^* + \sigma^* - \underline{c})$, and $\nu^S = 1 - \nu^N$. Hence,

$$\begin{aligned}
\frac{\partial\Delta}{\partial\nu^N} &= \left(1 - \frac{1}{\lambda^*}\right)\cdot(\overline{p}^* - \overline{c}) + \left(\nu^N + \frac{\nu^S}{\lambda^*}\right)\cdot\frac{\partial\overline{p}^*}{\partial\nu^N} - (\overline{p}^* + \sigma^* - \underline{c}) - \nu^N\cdot\frac{\partial\overline{p}^*}{\partial\nu^N} && (93)\\
&= \left(1 - \frac{1}{\lambda^*}\right)\cdot(\overline{p}^* - \overline{c}) - (\overline{p}^* + \sigma^* - \underline{c}) + \frac{\nu^S}{\lambda^*}\cdot\frac{\partial\overline{p}^*}{\partial\nu^N} && (94)\\
&= -\frac{1}{\lambda^*}\cdot\frac{1}{\nu^N}\cdot(\overline{p}^* - \overline{c}) + \frac{\nu^S}{\lambda^*}\cdot\frac{\partial\overline{p}^*}{\partial\nu^N}, && (95)
\end{aligned}$$

where we have inserted the equilibrium condition $\Delta(\lambda^*) = 0$ (which is equivalent to $\overline{p} + \sigma^* - \underline{c} = 1/\nu^N\cdot(\nu^N + \nu^S/\lambda^*)\cdot(\overline{p}^* - \overline{c})$) in the second line to obtain the third line. Recall from (27) that

$$\overline{p}^* - \overline{c} = \left(1 + \lambda^*\frac{\nu^N}{\nu^S}\right)\cdot\frac{1}{h(\hat{\theta}^*)}, \tag{96}$$

and observe that

$$\frac{\partial\overline{p}^*}{\partial\nu^N} = \frac{\lambda^*}{(1-\nu^N)^2}\cdot\frac{1}{h(\hat{\theta}^*)}. \tag{97}$$

After inserting (96) and (97) in (95) and simplifying terms, we obtain

$$\frac{\partial\Delta}{\partial\nu^N} = -\frac{1}{\lambda^* h(\hat{\theta}^*)\nu^N}, \tag{98}$$

which establishes (92) and completes the proof of part (i).

As to (ii). By (33), a straightforward calculation delivers that

$$\begin{aligned}
\frac{dW}{d\nu^N} &= -(\overline{q} - \overline{c} + \hat{\theta}^*) + \nu^S\cdot\frac{d\hat{\theta}^*}{d\nu^N} + \lambda^*\cdot(\Delta q - \Delta c) + \underline{q} - \underline{c} - s + \nu^N\cdot\frac{d\lambda^*}{d\nu^N}\cdot(\Delta q - \Delta c) && (99)\\
&\leq -(1-\lambda^*)\cdot(\Delta q - \Delta c) - \hat{\theta}^* - s, && (100)
\end{aligned}$$

where the inequality follows from $d\lambda^*/d\nu^N \leq 0$ by part (i), $d\hat{\theta}^*/d\nu^N \leq 0$, as

$$\frac{d\theta^*}{d\nu^N} = \frac{\partial\theta^*}{\partial\lambda}\cdot\frac{d\lambda^*}{d\nu^N} = -\frac{s}{(\lambda^*)^2\cdot g'(\hat{\theta}^*)}\cdot\frac{d\lambda^*}{d\nu^N} \leq 0, \tag{101}$$

and re–arranging terms.

Now, a revealed preference argument implies that the equilibrium utility of a sophisticated consumer, $U^S = \overline{q} - \overline{p}^* + \hat{\theta}^*$, is larger than the expected utility that he would obtain if he purchased the first superior product that he encounters, which is $\overline{q} - \overline{p}^* + \mathbb{E}(\theta) - \sigma^*$. Because $\mathbb{E}(\theta) = 0$, this implies that $\sigma^* \geq -\hat{\theta}^*$. Inserting this in (100) yields (recall $s = \sigma^* \cdot \lambda^*$)

$$\frac{dW}{d\,\nu^N} \leq -(1-\lambda^*) \cdot (\Delta q - \Delta c + \hat{\theta}^*), \tag{102}$$

and $\Delta q - \Delta c \geq -\hat{\theta}^*$ thus implies $dW/d\,\nu^N < 0$.

Now, observe that this condition is implied by the conditions (a) and (b) in the statement of the proposition. Indeed, (a) $\Delta q - \Delta c \geq \overline{\theta} - \underline{\theta}$ implies $\Delta q - \Delta c \geq -\hat{\theta}^*$, because $\hat{\theta}^* \geq \underline{\theta}$ and $\overline{\theta} > 0$, and (b) $\Delta q - \Delta c \geq \sigma^*$ implies $\Delta q - \Delta c \geq -\hat{\theta}^*$, because $\sigma^* \geq -\hat{\theta}^*$ as argued above. ■

Proof of Proposition 7 As argued in the text, it suffices to show that

$$\frac{d\overline{p}^*}{d\lambda} \leq 0 \quad \text{if} \quad \epsilon(\sigma^*) \geq 1 \tag{103}$$

Too see this, observe that (27), (28) and a straightforward calculation yield that

$$\frac{d\overline{p}^*}{d\lambda} = \frac{\partial \overline{p}^*}{\partial \lambda} + \frac{\partial \overline{p}^*}{\partial \sigma} \cdot \frac{\partial \sigma^*}{\partial \lambda} = \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} + \epsilon(\sigma^*) \cdot \frac{(\overline{p}^* - \overline{c})}{\sigma^*} \cdot (-\frac{s}{(\lambda^*)^2}). \tag{104}$$

Inserting (27) and simplifying terms yields that

$$\frac{d\overline{p}^*}{d\lambda} = \left(\frac{\nu^N}{\nu^S} - \epsilon(\sigma^*) \cdot (\frac{1}{\lambda^*} + \frac{\nu^N}{\nu^S})\right) \cdot \frac{1}{h(\hat{\theta}^*)}, \tag{105}$$

which establishes (103), as desired. ■

Proof of Lemma 4 The argument why a price floor improves total welfare is given in the main text.

As to consumer welfare, observe that by (6), (22) and (34) consumer welfare is given as

$$W^C = \nu^S \cdot (\overline{q} + \hat{\theta} - \overline{p}) + \nu^N \cdot (\underline{q} + \lambda \cdot \Delta q - (\overline{p} + \frac{s}{\lambda})), \tag{106}$$

where $\hat{\theta}$ and λ are the equilibrium market outcomes given a candid price $\overline{p}$ and are pinned down by (17) and (39). We want to show that

$$\frac{dW^C}{d\overline{p}} > 0 \quad \text{if} \quad \Delta q \text{ is sufficiently large.} \tag{107}$$

By (17) and Lemma A.1, $\hat{\theta}$ increases in λ, and by (39), λ increases in $\overline{p}$. Therefore, $\hat{\theta}$ increases in $\overline{p}$, and we infer that

$$\frac{dW^C}{d\overline{p}} \geq -\nu^S + \nu^N \cdot (\frac{d\lambda}{d\overline{p}} \cdot \Delta q - 1). \tag{108}$$

Differentiating (39) yields an expression for $d\lambda/d\overline{p}$, and, after simplifying terms, we obtain

$$\frac{dW^C}{d\overline{p}} \;\; \geq \;\; -1+\frac{\nu^S}{\Delta c}\cdot\Delta q, \tag{109}$$

which is positive if Δq is sufficiently large, as desired. ■

Proof of Lemma 5 As to total welfare, as argued in the text, it suffices to show that the share of candid firms increases in response to a minimum quality standard. To see this, observe that the difference in candid and deceptive profits as given in (24) is independent of $\underline{q}$. The effect of a minimum quality standard on the share of candid firms is therefore given by $d\lambda^*/d\underline{c} = -(\partial\Delta/\partial\underline{c})/(\partial\Delta/\partial\lambda)$. Because $\partial\Delta/\partial\lambda<0$ by (65),

$$\frac{\partial\Delta}{\partial\underline{c}} \;\; = \;\; \nu^N \tag{110}$$

implies $d\lambda^*/d\underline{c}\geq 0$, as desired.

As to consumer welfare, as is argued in the text, if suffices to show that $d\overline{p}^*/d\lambda\leq 0$ if $\epsilon(\sigma^*)\geq 1$. But this follows immediately from (103). ■

Proof of Lemma 6 First, we show that there is no pooling equilibrium. To the contrary, suppose that there is a segmented market equilibrium such that all firms charge p^*, and the share of candid firms is $\lambda^*\in(0,1)$. In a pooling equilibrium, an uninformed consumer cannot distinguish between superior and inferior products and purchases from the first firm he visits in equilibrium. Because a consumer's reservation utility equals his expected equilibrium utility, the reservation utility of an uninformed consumer is therefore given by

$$\hat{U}^N=\lambda^*\cdot\overline{q}+(1-\lambda^*)\cdot\underline{q}-p^*-s. \tag{111}$$

Now, an uninformed consumer expects any product offered at p to supply the utility

$$U(p)=\hat{\lambda}(p)\cdot\overline{q}+(1-\hat{\lambda}(p))\cdot\underline{q}-p. \tag{112}$$

Therefore, he purchases a deceptive product offered at p^*+dp, because $U(p^*+dp)>\hat{U}^N$ for small dp by consistency and continuity of $\hat{\lambda}$. As a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, it would be profitable for it to raise its price marginally – a contradiction of the optimality of prices in equilibrium.

Second, we show that there is no separating equilibrium. Suppose the contrary and consider first the case $\underline{p}^*>\overline{p}^*$. We derive a contradiction by arguing that a candid firm can profitably deviate to a higher price. Indeed, because in a separating equilibrium uninformed consumers learn a firm's quality, for a deceptive firm to derive demand from uninformed consumers in equilibrium, uninformed consumers must prefer purchasing a inferior product at $\underline{p}^*$ to searching for a superior product:

$$(\underline{p}^* - \overline{p}^*) + \Delta q \leq \frac{s}{\lambda^*}. \tag{113}$$

To see the inequality, note that s/λ^* are the expected costs to find a superior product not only for a sophisticated consumer but also for an uninformed consumer, as he recognizes superior products in a separating equilibrium. Now, (113) implies $\Delta q < s/\lambda^*$ because $\underline{p}^* > \overline{p}^*$ by assumption. By assumption (4), we thus have $\overline{\theta} - \underline{\theta} < s/\lambda^*$. Moreover, in a segmented market equilibrium a sophisticated consumer only purchases superior products and, as before, his search rule is characterized by a reservation match-value $\hat{\theta}^*$ that is given by (8). By (8) and Lemma A.1, $\overline{\theta} - \underline{\theta} < s/\lambda^*$ implies $\hat{\theta}^* < \underline{\theta}$. In equilibrium, a sophisticated consumer therefore strictly prefers to purchase a superior product at $\overline{p}^*$, irrespective of product fit, instead of continuing to search. Note, however, that the same must be true for uninformed consumers, because they purchase in equilibrium the deceptive product which is more expensive and provides a lower quality. Thus, because consumer beliefs $\hat{\lambda}$ are continuous, the entire demand of a candid firm is (locally) inelastic at $\overline{p}^*$ such that it is profitable for a candid firm to increase its price – contradicting that $\overline{p}^*$ is an equilibrium price.

Next consider the case that $\underline{p}^* < \overline{p}^*$. Let ξ be the probability with which an uninformed consumer purchases a product that is offered at $\overline{p}^*$. Because an uninformed consumer purchases superior products in a segmented market equilibrium, $\xi \in (0,1]$. If $\xi = 1$, then a deceptive firm could charge $\overline{p}^*$, and any visiting uninformed consumers would purchase its product. Because $\underline{p}^* < \overline{p}^*$ and because a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, this would constitute a profitable deviation—a contradiction. Therefore, $\xi \in (0,1)$, and an uninformed consumer is indifferent between purchasing a product offered at $\overline{p}^*$ and continuing to search. Hence, because in a separating equilibrium, an uninformed consumer expects an product offered at $\overline{p}^*$ to deliver utility $\overline{q} - \overline{p}^*$, his reservation is:

$$\hat{U}^N = \overline{q} - \overline{p}^*. \tag{114}$$

Because a naive consumer purchases superior and deceptive products in equilibrium, it is optimal for him to purchase at the first firm he visits. His reservation utility therefore also satisfies

$$\hat{U}^N = \lambda^* \cdot (\overline{q} - \overline{p}^*) + (1 - \lambda^*) \cdot (\underline{q} - \underline{p}^*) - s, \tag{115}$$

and by (114), we have

$$\hat{U}^N = (\underline{q} - \underline{p}^*) - \frac{s}{1 - \lambda^*}. \tag{116}$$

In a separating equlibrium, an uninformed consumer expects a product offered at $\underline{p}^*$ to supply the utility

$$U(\underline{p}^*) = \underline{q} - \underline{p}^*. \tag{117}$$

By (116) and (117), an uninformed consumer therefore strictly prefers to purchase a product offered at $\underline{p}^*$ instead of continuing to search. As a consequence, because consumer beliefs are continuous in prices, a deceptive firm could raise its price marginally and would not loose any demand from uninformed consumers. As a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, such a deviation would be profitable, contradicting the assumption that $\underline{p}^*$ is an equilibrium price. ■

Proof of Lemma 7 First, we argue that there is a segmented market equilibrium with $\underline{p}^* > \overline{p}^*$ if ε and search costs s are sufficiently small. Second, we argue that the share of candid firms vanishes as search costs vanish.

Before, observe that, by Bayes' rule in a segmented market equilibrium, the expected quality of a product, conditional on a good (resp. bad) signal is

$$\mathbb{E}(q|\overline{z}) = \frac{\lambda(\frac{1}{2}+\varepsilon)\overline{q}+(1-\lambda)(\frac{1}{2}-\varepsilon)\underline{q}}{\lambda(\frac{1}{2}+\varepsilon)+(1-\lambda)(\frac{1}{2}-\varepsilon)}, \quad \left(\text{resp.} \quad \mathbb{E}(q|\underline{z}) = \frac{\lambda(\frac{1}{2}-\varepsilon)\overline{q}+(1-\lambda)(\frac{1}{2}+\varepsilon)\underline{q}}{\lambda(\frac{1}{2}-\varepsilon)+(1-\lambda)(\frac{1}{2}+\varepsilon)}\right).$$

Observe that $\mathbb{E}(q|\overline{z}) \geq \mathbb{E}(q) \geq \mathbb{E}(q|\underline{z})$, and for each δ there exists ε such that $\mathbb{E}(q|\overline{z})-\mathbb{E}(q|\underline{z}) < \delta$.

Claim *For sufficiently small ε and sufficiently small search costs s, there is a solution $\overline{p}^* \geq 0$, $\underline{p}^* \geq 0$, $\hat{p}^* \geq 0$, $\hat{\theta}^* \in (\underline{\theta},\overline{\theta})$ and $\lambda^* \in (0,1)$ to the equation system (17), (19-21) and (replacing (18))*

$$\hat{p}^* = \lambda^* \cdot \overline{p}^* + (1-\lambda^*) \cdot \underline{p}^* + s - (\mathbb{E}(q) - \mathbb{E}(q|\underline{z})). \tag{118}$$

The solution displays $\underline{p}^* > \overline{p}^*$ and describes a segmented market equilibrium outcome.

To prove the claim, we first show that the equation system (17), (19-21), (118) characterizes the best responses of all agents, given a (candidate) segmented market equilibrium outcome $(\lambda^*, \overline{p}^*, \underline{p}^*)$.

Indeed, as to a naive consumer's optimal best response, (20) and (118) imply

$$\underline{p}^* \quad = \quad \overline{p}^* + \frac{s-(\mathbb{E}(q)-\mathbb{E}(q|\underline{z}))}{\lambda^*}. \tag{119}$$

For sufficiently small ε, we have $s > \mathbb{E}(q) - \mathbb{E}(q|\underline{z})$, and thus $\overline{p}^* < \underline{p}^*$. Moreover, by (20), we have $\underline{p}^* = \hat{p}^*$. Together, this implies $\overline{p}^* < \underline{p}^* \leq \hat{p}^*$. Now, if it is optimal for a naive consumer to purchase at the first firm he visits (even if he receives a bad signal), then his reservation utility is given by (10). In this case, (118) is a complete characterization of a naive consumer's optimal search rule (his reservation utility), because (118) denotes the maximal price that he is willing to pay for a product upon receiving a bad signal given (10), as can be shown straightforwardly. To see that it is indeed optimal for a naive consumer to purchase at the first firm he visits, it suffices to verify that $\overline{p}^* < \underline{p}^* \leq \hat{p}^*$ holds, which is the case, as was shown before.

As to a sophisticated consumer's optimal best response, as argued in the proof of Lemma 2, $\underline{p}^* > \overline{p}^*$ implies that a sophisticated consumer does not purchase deceptive products, and (17) characterizes his optimal search rule, as before.

As to a candid firm's behavior, for prices below $\hat{p}^*$, profits of candid firms are given by (14). Because $\underline{p}^* > \overline{p}^*$ and $\underline{p}^* = \hat{p}^*$, (14) is differentiable at $\overline{p}^*$ and (19) is the first order condition for profit maximization of candid firms, as before.

As to a deceptive firm's behavior, for a deceptive firm it is optimal to charge $\hat{p}$, because its demand from naive consumer is inelastic up to $\hat{p}$ (leaving the demand from sophisticated consumer aside). If a deceptive firm charged a price that exceeds $\hat{p}$, it would only derive demand from naive consumer who receive a good signal about its quality. At most a deceptive firm could raise its price (and hence its mark–up) by $\mathbb{E}(q|\overline{z}) - \mathbb{E}(q|\underline{z})$, so as to still derive demand from these consumers, whereas it would loose the demand of all naive consumers who receive a bad signal which amounts to a mass of $\nu^N \cdot (1/2 - \varepsilon)$. For ε sufficiently small, a deceptive firm hence looses almost half of its demand, whereas it only increases its mark–up by less than some δ. Because $\underline{p}^* > \overline{p}^*$, the mark–up of a candid firm is bounded away from zero in equilibrium, which implies that such a deviation is not profitable for sufficiently small ε. Finally, (21) ensures that firms in the two segments earn the same profits.

A proof why the first order conditions is sufficient for profit maximization for a candid firm and why it not profitable for a deceptive firm to lower its price by so much that it derives demand from sophisticated consumers follows along the lines of the proof of Lemma 2. Moreover, an argument why there is a solution to the above the equation system for sufficiently small search costs follows along the lines of Proposition 1. We omit these details, and this completes the proof of the claim.

As the second step of the proof, we now show that the share of candid firms vanishes as search costs do. Because $\underline{p}^* > \overline{p}^*$, the deceptive mark–up is bounded away from zero. This implies that deceptive profits are bounded away from zero, because the demand of a deceptive firm is ν^N. Now, suppose that λ^* would not vanish as search costs get small. Then, λ^* would be bounded from below and effective search costs would vanish as search costs get small. This would imply that a candid firm's mark–up and profits vanish, because, as before, a candid firm's price is given by (19). This contradicts that firms in the two segments earn the same profit. ■

B Appendix

In this appendix, we show that if the distribution f^E of match-values is given by (31), then there an open set of parameters $s, \overline{q}, \underline{q}, \underline{c}, \overline{c}, \nu^N$ such that there is a segmented market equilibrium with

$\hat{\theta}^* < 0$ and

$$\lambda^* = \frac{\frac{\nu^S}{\nu^N} - s}{\Delta c - 1} \quad \text{and} \quad \overline{p}^* - \overline{c} = \frac{\Delta c - s\frac{\nu^N}{\nu^S}}{\Delta c - 1}. \tag{120}$$

Note that the cumulative distribution function of f^E is

$$F^E(\theta) = \begin{cases} 1 - e^{-(\theta+1)} & \theta < 0 \\ 1 + e^{-1} \cdot (\theta - 1) & \theta \geq 0 \end{cases} \tag{121}$$

and satisfies $h^E(\theta) = 1$ for $\theta \leq 0$.

Moreover, the corresponding function g^E as defined by (7) is given by[51]

$$g^E(z) = \begin{cases} \frac{1}{2e} \cdot (2e^{-z} - 1) & z < 0 \\ \frac{1}{2e} \cdot (1 - z)^2 & z \geq 0. \end{cases} \tag{125}$$

We now show that provided $\hat{\theta}^* < 0$, the formulae in (120) satisfy the equilibrium conditions (19-21).[52] In fact, $\hat{\theta}^* < 0$ implies $h^E(\hat{\theta}^*) = 1$, and thus $\Delta(\lambda^*) = 0$ becomes equivalent to

$$\frac{\nu^S}{\lambda^*} \cdot \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) - \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c\right) = 0. \tag{126}$$

Rearranging terms then yields the desired expression for λ^*. Inserting $h^E(\theta^*) = 1$ and λ^* from (120) into equation (19) yields

$$\overline{p}^* - \overline{c} = 1 + \frac{1 - s\frac{\nu^N}{\nu^S}}{\Delta c - 1}, \tag{127}$$

[51] To see this, note that (7) implies for $z < 0$:

$$g^E(z) = \int_z^0 (\theta - z)\, e^{-(\theta+1)}\, d\theta + \int_0^1 (\theta - z)\, e^{-1}\, d\theta. \tag{122}$$

Integration by parts yields

$$\int_z^0 (\theta - z)\, e^{-(\theta+1)}\, d\theta \quad = \quad -(\theta - z)\, e^{-(\theta+1)}\Big|_z^0 + \int_z^0 e^{-(\theta+1)}\, d\theta = (z-1)\, e^{-1} + e^{-(z+1)}. \tag{123}$$

Moreover, $\int_0^1 (\theta - z)\, e^{-1}\, d\theta = e^{-1} \cdot (\frac{1}{2} - z)$. Inserting this in (122) yields the first line in (125.)

For $z \geq 0$, g^E corresponds to the second line in (125), because by (7):

$$g^E(z) = \int_z^1 (\theta - z)\, e^{-1}\, d\theta = e^{-1} \cdot (\frac{1}{2}\theta^2 - z\theta)\Big|_z^1 = e^{-1} \cdot (\frac{1}{2} - z + \frac{1}{2}z^2) = \frac{1}{2e} \cdot (1 - z)^2. \tag{124}$$

[52] Strictly speaking, it is not a priori clear that with the density f^E the equilibrium conditions (17-21) are actually sufficient for equilibrium existence. The reason is that f^E is not globally log-concave (which we use in our sufficiency proof). However, for the specific case that $f = f^E$, the sufficiency proof still goes through with minor changes. We omit the details.

which is equivalent to the expression in (120).

To complete the proof, we show that there are parameters so that for λ^* as given in (120), we have that $\lambda^* \in (0,1)$, and the equilibrium match-value $\hat{\theta}^*$ which is given as $\hat{\theta}^* = (g^E)^{-1}(s/\lambda^*)$ by (17) indeed satisfies $\hat{\theta}^* < 0$.

To see this, let $\nu^S/\nu^N = 1/e$, $s = 1/(2e)$ and $\Delta c = 1 + 1/e$. By a straightforward but tedious calculation, we obtain

$$\lambda^* = \frac{1}{2}, \quad \hat{\theta}^* = -\ln(3/2), \quad \overline{p}^* = \frac{e}{2} + 1, \quad \underline{p}^* = \frac{e}{2} + 1 + \frac{1}{e}. \tag{128}$$

Finally, note that because the equilibrium values are continuous in the parameters, if we perturb the parameters slightly, this does not upset the (strict) inequalities $\lambda^* \in (0,1)$ and $\hat{\theta}^* < 0$. This establishes the claim. ■

References

Anderson, S. P. and R. Renault (1999). Pricing, Product Diversity, and Search Costs: a Bertrand-Chamberlin-Diamond Model. *The RAND Journal of Economics 30*(4), 719–735.

Armstrong, M. and Y. Chen (2009). Inattentive Consumers and Product Quality. *Journal of the European Economic Association 7*, 411–422.

Armstrong, M. and J. Vickers (2012). Consumer Protection and Contingent Charges. *Journal of Economic Literature 50*(2), 477–493.

Armstrong, M. and J. Zhou (2011). Paying for Prominence. *Economic Journal 121*, 368–395.

Ausubel, L. M. (1991). The Failure of Competition in the Credit Card Market Author. *The American Economic Review 81*(1), 50–81.

Bakos, Y. J. (1997). Reducing Buyer Search Costs: Implications for Electronic Marketplaces. *Management Science 43*(12), 1676–1692.

Bar-Isaac, H., G. Caruana, and V. Cuñat (2012). Search, Design, and Market Structure. *American Economic Review 102*(2), 1140–1160.

Cachon, G. P., C. Terwiesch, and Y. Xu (2008, may). On the Effects of Consumer Search and Firm Entry in a Multiproduct Competitive Market. *Marketing Science 27*(3), 461–473.

Choi, M., A. Y. Dai, and K. Kim (2017). Consumer Search and Price Competition. *Iowa Mimeo*.

Diamond, P. A. (1971). A model of price adjustment. *Journal of Economic Theory 3*(2), 156–168.

Ellison, G. and S. F. Ellison (2009). Search, Obfuscation, and Price Elasticities on the Internet. *Econometrica 77*(2), 427–452.

Fishman, A. and N. Levy (2015). Search Costs and Investment in Quality. *Journal of Industrial Economics 63*(4), 625–641.

Gabaix, X. and D. Laibson (2006). Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets. *The Quarterly Journal of Economics 121*(2), 505–540.

Gamp, T. (2016a). Guided Search. *Bonn Mimeo*.

Gamp, T. (2016b). Search, Differentiated Products and Obfuscation. *Bonn Mimeo*.

Gurun, U. G., G. Matvos, and A. Seru (2016). Advertising Expensive Mortgages. *Journal of Finance 71*(5), 2371–2416.

Heidhues, P., J. Johnen, and B. Koszegi (2017). Browsing versus Studying Offers. *DICE Mimeo*.

Heidhues, P., B. Köszegi, and T. Murooka (2017). Inferior Products and Profitable Deception. *Review of Economic Studies 84*(1), 323–356.

Hortacsu, A. and C. Syverson (2004). Product Differentiation, Search Costs, and Competition in the Mutual Fund Industry: A Case Study of S&P 500 Index Funds. *Quarterly Journal of Economics 119*(2), 403–456.

Janssen, M. C. and J. L. Moraga-González (2004). Strategic Pricing, Consumer Search and the Number of Firms. *The Review of Economic Studies 71*(4), 1089–1118.

Janssen, M. C. and S. Shelgia (2015). Consumer search and double marginalization. *The American Economic Review 105*(6), 1683–1710.

Jehiel, P. (2005). Analogy-based expectation equilibrium. *Journal of Economic Theory 123*(2), 81–104.

Kuksov, D. (2004, sep). Buyer Search Costs and Endogenous Product Design. *Marketing Science 23*(4), 490–499.

Larson, N. (2013). Niche products, generic products, and consumer search. *Economic Theory 52*(2), 793–832.

Lee, J. and J. M. Hogarth (2000). Consumer information search for home mortgages: who, what, how much, and what else? *Financial Services Review 9*(3), 277–293.

Mayer, C. J. and K. Pence (2008). Subprime mortgages: what, where, and to whom? *NBER Working Paper 14083*.

McCall, J. J. (1970). Economics of Information and Job Search. *The Quarterly Journal of Economics 84*(1), 113–126.

Moraga-González, J. L., Z. Sándor, and M. R. Wildenbeest (2017). Prices and Heterogeneous Search Costs. *Rand Journal of Economics (forthcoming)*.

OFT (2008). Personal current accounts in the UK: An OFT market study. *Office of Fair Trading*.

Piccione, M. and R. Spiegler (2012). Price Competition under Limited Comparability. *Quarterly Journal of Economics 127*(1), 97–135.

Scharfstein, D. S. and A. Sunderam (2013). Concentration in Mortgage Lending, Refinancing Activity and Mortgage Rates. *NBER Working Paper 19156*.

Sirri, E. R. and P. Tufano (1998). Costly Search and Mutual Fund Flows. *The Journal of Finance 53*(5), 1589–1622.

Spiegler, R. (2006). Competition over Agents with Boundedly Rational Expectations. *Theoretical Economics 1*, 207–231.

Stahl, D. O. I. (1989). Oligopolistic Pricing with Sequential Consumer Search. *The American Economic Review 79*(4), 700–712.

Stango, V. and J. Zinman (2009). What Do Consumers Really Pay on Their Checking and Credit Card Accounts? Explicit, Implicit, and Avoidable Costs. *The American Economic Review 99*(2), 424–429.

Stango, V. and J. Zinman (2014). Limited and Varying Consumer Attention: Evidence from Socks to the Salience of Bank Overdraft Fees. *Review of Financial Studies 27*(4), 990–1030.

Taylor, G. (2017). Raising search costs to deter window shopping can increase profits and welfare. *RAND Journal of Economics 48*(2), 387–408.

Varian, H. R. (1980). A Model of Sales. *The American Economic Review 70*(4), 651–659.

Vickers, J., M. Armstrong, and J. Zhou (2009). Consumer Protection and the Incentive to Become Informed. *Journal of the European Economic Association 7*(2-3), 399–410.

Wolinsky, A. (1986). True Monopolistic Competition as a Result of Imperfect Information. *The Quarterly Journal of Economics 101*(3), 493–512.

Woodward, S. E. and R. E. Hall (2010). Consumer Confusion in the Mortgage Market : Evidence of Less than a Perfectly Transparent and Competitive Market. *American Economic Review: Papers & Proceedings 100*(May), 511–515.